Math 454 Section 001

Fall 2009

Boundary Value Problems for PDEs

This advanced course is devoted to the solutions and behaviors of second-order linear partial differential equations (PDEs) in the context of (initial-)boundary value problems. Emphasis is on concepts and calculation with worked examples throughout the course.

Students will learn important techniques for solving PDEs: separation of variables, Fourier series/transform, Sturm-Liouville eigenvalue problems, eigenfunction expansion, Green's functions, etc. In particular, students will gain understandings, both qualitative and quantitative, of such classical PDEs as heat equation, wave equation and their variations. Applications are wide and diverse, ranging from physical sciences (fluid dynamics, electromagnetism, ...) to biological sciences to social sciences, though we will use examples mainly from Physics.

Instructor	Bin Cheng (4839 EH. email: bincheng)
Lectures	TuTh 12-130pm at 2166 DOW
Office hours	TuTh 130-230pm at the lounge next to 2166 DOW, W 130-230pm at 4839 EH
Textbook	Applied partial differential equations, 4th edition *by* Richard Haberman
Website	http://www.umich.edu/~bincheng/Math454F2009/
Prerequisite	Math 216, 256, 286 or 316

Attendance (10%): Mandatory.

Homework (40%): Assignments can be downloaded from the website (see above). Students then spend one or two weeks to work on them. Group discussion is encouraged but everyone must submit his/her own write-up.

* A random selection of problems are graded.
* Solutions are available about one week after due time.
* Extension (1-5 days) may be arranged upon written request.

Midterm I (15%): Closed book. Monday 6-8pm, Oct/12. Room TBA.

Midterm II (15%): Take home. Week 11 (one week before the holiday).

Final (20%): Closed book. Tuesday 4-6pm, Dec/22. Room TBA.

List of materials covered (tentative)

- Week 1. (9/8, 9/10) Handout, Sec. 1.1 — 1.4
  Preliminary. Introduction to PDEs. Derivation of one-dimensional heat equation.
- Week 2. (9/15, 9/17) Sec. 2.1 — 2.4
  Linearity and Principle of Superposition. Solving 1D heat equation using separation of variables. Fourier series: coefficient formula.
- Week 3. (9/22, 9/24) Sec. 2.5, 3.1 — 3.2
  Laplace equations with boundary conditions. Fourier series: convergence.
- Week 4. (9/29, 10/1) Sec. 3.3 — 3.5
  Fourier sine and cosine series. Term-by-term differentiation and integration. Generalized Fourier series.
- Week 5. (10/6, 10/8) Sec. 4.1 — 4.4
  One-dimenstional wave equation: derivation and solution.
- Week 6. (10/13, 10/15) Sec. 5.1 — 5.5
  One-dimensional Sturm-Liouville problem. Self-adjoint operator. (Weighted) orthogonality of eigenfunctions. Applications in PDE.
- Week 7. (10/22) Sec. 5.6 — 5.7. Rayleight Quotient.
- Week 8. (10/27, 10/29) Sec. 5.8 — 5.9
  Boundary conditions. Approximation of eigenvalues and eigenfunctions.
- Week 9. (11/3, 11/5) Sec. 7.1 — 7.4
  PDE in higher dimensions. Two-dimensional wave equation.
- Week 10. (11/10, 11/12) Sec. 7.5 — 7.7
  Multi-dimensional Sturm-Liouville problem. Bessel's functions.
- Week 11. (11/17, 11/19) Sec. 7.9 — 7.10
  Laplace's equation in a circular cylinder. Legendre polynomials.
- Week 12. (11/24) Sec. 8.1 — 8.4. Method of eigenfunction expansion.
- Week 13. (12/1, 12/3) Sec. 9.5. Green's function for Poisson's equation
- Week 14. (12/8, 12/10) Sec. 10.1 — 10.4, Handout
  Introduction to Fourier transform. Review.