Math 354 Review

Final exam: 4-6pm, April 21, Monday at B261 East Hall
No references other than an A4 page of notes

Textbook materials covered:
Chapter 1. §1.1,1.3
Chapter 2. §2.1 – 2.5
Chapter 3. §3.1, 3.2, 3.3(except completeness), 3.4(except the dominated convergence theorem), 3.5
Chapter 4. § 4.2#, 4.3#, 4.4(except the Dirichlet problem in polar coordinates)
Chapter 5. Introduction, §5.1(except power series solution), 5.4*, 5.5*
Chapter 6. §6.1,6.2*,6.4*,6.5*,6.6(only Haar functions and wavelets)
Chapter 7. Introduction, §7.1(only theorem 7.1, 7.2), 7.2, 7.3(except the sampling theorem and after), 7.5*, 7.6#(only the DFT and FFT)

**: partially, usually the first part.*

#: we used different approaches in class than the textbook.

Fourier Series.

1. Definitions for 2π-periodic functions and general periodic functions.
2. Pointwise convergence of F.S. for piecewise continuous functions.
3. Term-by-term differentiation and integration.
4. Smoothness and decay of Fourier coefficients.
5. Fourier sine and cosine series.

Orthogonal Sets of Functions in L^2.

1. Inner products in vector spaces and function spaces.
2. The Cauchy-Schwartz inequality. The Triangle inequality. The Pythagorean theorem.
3. Convergence in norm.
4. Orthonormal basis. Generalized Fourier series.
5. Best L^2 approximation.
6. Sturm-Liouville problem.
– Self-adjoint operator (with suitable boundary conditions).
– Eigenvalues and eigenfunctions.
– Orthonormal bases consisting of eigenfunctions.

Partial Differential Equations (on bounded spatial domains).

1. Separation of variables.
– Sturm-Liouville problems with boundary conditions.
– Family of special solutions.
– General solutions. Initial conditions.

2. Heat flow problems on bounded intervals.
– Time-dependent and steady-state solutions.
– Dirichlet and von Neumann boundary conditions.
– Asymptotic behavior.
– Inhomogeneous boundary conditions.
– Inhomogeneous source terms. Variation of parameters.

3. Wave equations.
– Same issues as above.
– Initial conditions on u and u_t.

4. Laplace equations on a square.

Examples of Orthogonal Bases.

1. Bessel functions.
– Bessel's equations. Laplace operator in polar coordinates.
– First and second kinds of Bessel functions.

2. Orthogonal polynomials.
– Properties of $\{p_n\}$. Degrees, orthogonality, $span\{p_n\}_{n=0}^N = ...$
– Rodrigues formulas.
– Weighted L^2 inner products and norms.
– Legendre, Hermite, Laguerre polynomials. The Sturm-Liouville problems they solve.

3. Wavelets. Definitions. Localization.

Fourier Transforms (continuous and discrete).

1. Definitions.

2. Convolution.

3. Properties. (Table 2 on page 223) Translation, dilation, differentiation, convolution ...

4. Applications in PDEs on infinite spatial domains.
– Heat equation and heat kernel.
– General techniques (PDE —(F.T.)—>ODEs).

5. Multivariable Fourier transform.

6. Discrete Fourier transform.
– Fourier matrix. Orthonormal basis of $\mathbf{C}^N$.
– Basic idea of fast Fourier transform.

7. Applications in signal processing.
– System function. Impulse response. Effects on frequencies.
– Ideas of smoothing with filters (e.g. the heat kernel).