Math 354 Handout 1 – Preliminary

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- Complex numbers and functions

$$i \cdot i = -1$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}, \qquad \sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

$$(e^{i\theta})^n = e^{in\theta}$$

$$e^{i\alpha}e^{i\beta} = e^{i(\alpha+\beta)}, \qquad \frac{e^{i\alpha}}{e^{i\beta}} = e^{i(\alpha-\beta)}$$

$$\overline{a+bi} = a - bi, \qquad \overline{e^{i\theta}} = e^{-i\theta} \qquad \text{here } a, b, \theta \in \mathcal{R}$$

$$|z| = \sqrt{z\bar{z}}, \qquad |a+bi| = \sqrt{a^2+b^2}, \qquad |re^{i\theta}| = r$$

- Series

  We are here concerned with the convergence of series $\sum_{n=0}^{\infty} a_n$.

  Definition: $\sum a_n$ converges if the partial sum $S_N := \sum_{n=0}^{N}$ approaches a finite limit as $N \to \infty$.

  - Comparison test. If $|a_n| \le b_n$ and $\sum b_n$ converges, then $\sum a_n$ converges.
  - Ratio test.
  - Root test.
  - Integral test.

- p-test. $\sum n^{-p}$ converges if $p > 1$ and diverges if $p \leq 1$.
- Alternating series.

- (Ordinary) differential equations
  - Seperation of variables.
  - Integrating factor.
  - $u'' + k^2 u = 0$.
  - Initial value problems and boundary value problems.
  - Qualitative properties of equations without explicit solutions.

- Vector space (from linear algebra)
  - Linear independence.
  - Basis.
  - Inner product.
  - Orthogonality.
  - Pythagorean theorem.