Math 354 Homework 4

Due at 10am Friday, Feb. 22th (before Spring Break)

Problem 1. (§ 3.2 # 1) Show that

$$\left\{ \sqrt{\frac{2}{l}} \sin \left[(n - \frac{1}{2}) \frac{\pi x}{l} \right] \right\}_1^\infty$$

is an orthonormal set in $L^2([0, l])$. You may use identity

$$2 \sin \alpha \sin \beta = \cos(\alpha - \beta) - \cos(\alpha + \beta).$$

Problem 2. (§ 3.2 # 3 revised) Consider $L^2([-1, 1])$ and $f_0(x) = 1$, $f_1(x) = x$.

a) Compute the L^2 norms of f_0, f_1.

b) Show that f_0, f_1 are orthogonal with respect to the $L^2([-1, 1])$ inner product.

c) Find constants a, b such that $f_2(x) = x^2 + ax + b$ is orthogonal to both f_0 and f_1.

d) Construct an orthonormal set using f_0, f_1, f_2.

Problem 3. Consider a <u>sequence</u> of functions $\{f_n(x)\}_1^\infty$ where

$$f_n(x) = x^n \text{ for } x \in [0, 1].$$

Show that $f_n \to 0$ in L^2 norm but not pointwise.

Problem 4. (§ 3.3 # 1) Let $f_n \to f$ in norm. Show that for any g in the same space,

$$\langle f_n, g \rangle \to \langle f, g \rangle.$$

Hint: apply the Cauchy-Schwartz inequality to $\langle f_n - f, g \rangle$.