# Math 354 Homework 2

## **Part 1 of 2**

Due at 10am Wednesday, January 30th

**Problem 1** (§ 2.2 #2) To what values do the series in entries 6,7,12 and 18 of Table 1 converge at the discontinuity points? These Fourier series are computed from the following functions (indeed, their $2\pi$-periodic extensions). Plot graphs of these functions if it is necessary.

entry 6: $f(\theta) = \begin{cases} -1, & \theta \in (-\pi, 0) \\ 1, & \theta \in (0, \pi) \end{cases}$

entry 7: $f(\theta) = \begin{cases} 0, & \theta \in (-\pi, 0) \\ 1, & \theta \in (0, \pi) \end{cases}$

entry 12: $f(\theta) = \begin{cases} (2a)^{-1}, & |\theta| < a \\ 0, & a < |\theta| < \pi \end{cases}$

entry 18: $f(\theta) = e^{b\theta}, \quad \theta \in (-\pi, \pi)$

**Problem 2** (§ 2.3 #3) Use entry 8 of Table 1 (that is $|\sin\theta| = \frac{2}{\pi} - \frac{4}{\pi}\sum_0^\infty \frac{\cos 2n\theta}{4n^2-1}$) and Theorem 2.1 to show the following formulas are true,

$$\sum_1^\infty \frac{1}{4n^2-1} = \frac{1}{2}, \qquad \sum_1^\infty \frac{(-1)^{n+1}}{4n^2-1} = \frac{\pi-2}{4}.$$

**Problem 3** Consider a $2\pi$-periodic function $f(\theta)$. Let $F(\theta) := \int_0^\theta f(x)\,dx$ and $c_0 := \frac{1}{2\pi}\int_{-\pi}^{\pi} f(\theta)\,d\theta$. Show that $F(\theta) - c_0\theta$ is also $2\pi$-periodic.

**Problem 4** (§ 2.3 #2) Starting from entry 16 of Table 1 (that is $\theta^2 = \frac{\pi^2}{3} + 4\sum_1^\infty \frac{(-1)^n}{n^2}\cos n\theta$ for $\theta \in [-\pi, \pi]$) and using Theorem 2.4, show that

b) $\theta^4 - 2\pi^2\theta^2 = 48\sum_1^\infty \dfrac{(-1)^{n+1}\cos n\theta}{n^4} - \dfrac{7\pi^4}{15}$ for $\theta \in [-\pi, \pi]$.

c) $\sum_1^\infty \dfrac{1}{n^4} = \dfrac{\pi^4}{9}$.

**Problem 5** (§ 2.3 #7) How smooth are the following functions? That is, how many derivatives can you guarantee them to have?

a) $f(\theta) = \sum_{-\infty}^{\infty} \dfrac{e^{in\theta}}{n^{13.2} + 2n^6 - 1}$.

b) $f(\theta) = \sum_0^{\infty} \dfrac{\cos n\theta}{2^n}$.

c) $f(\theta) = \sum_0^{\infty} \dfrac{\cos 2^n\theta}{2^n}$.

Justify your answers!