

# Math 354      Homework 1

Due at 10am Wednesday, January 16th

**Problem 1** (§ 1.1 #1) Show that  $u(x, t) = t^{-1/2} \exp\left(-\frac{x^2}{4kt}\right)$  satisfies the heat equation  $u_t = ku_{xx}$  for  $t > 0$ .

**Problem 2** (§ 1.3 #1) Derive a pair of ODEs from the following PDE by separation of variables.

a)  $yu_{xx} + u_y = 0$

**Problem 3** Use the results in the class to solve

$$\begin{cases} u_t = 10u_{xx} \\ u(0, t) = u(\pi, t) = 0 \\ u(x, 0) = 2 \sin 2x - 3 \sin 5x \end{cases} \quad \text{for } 0 \leq x \leq \pi \text{ and } t > 0.$$

**Problem 4** (§ 2.1 Table 1) Show that the following series are indeed the Fourier series of corresponding functions.

$$\#15 \quad f(\theta) = \begin{cases} a^{-2}(a - |\theta|) & \text{for } |\theta| < a \\ 0 & \text{for } a < |\theta| < \pi \end{cases}$$

$$f(\theta) = \frac{1}{2\pi} + \frac{2}{\pi} \sum_1^{\infty} \frac{1 - \cos na}{n^2 a^2} \cos n\theta.$$

$$\#19 \quad f(\theta) = e^{b\theta} \quad (0 < \theta < 2\pi).$$

$$f(\theta) = \frac{e^{2\pi b} - 1}{2\pi} \sum_{-\infty}^{\infty} \frac{e^{in\theta}}{b - in}.$$