

Things you should know.

**1, M 12/13, 9:50am - 11:40am. Total score 200 points.
Location: HERE! No personal notes but a table of
necessary formulas will be given.**

2, HW (25%). 11 scores in total (HW4 has two parts). Calculate your percentages of each HW and average the highest 8 percentages. Multiply it with 25%. Let A denote this number.

3, Tests. 3 scores with curving. Calculate your percentages of each test and average the highest 2 percentages. Multiply it with 40%. Let B denote this number.

4. Quizzes. Calculate your percentages of each quiz and average the highest 3 percentages. Multiply it with 5%. Let C denote this number.

5. Final. Score will be curved so that the class average is 80. Multiply your percentage with 30%. Let D denote this number.

$A+B+C+D=\text{total}$. Check the syllabus for grade cut-offs. Plus and minus will be determined at $2/3$ and $1/3$ or each bracket.

Mathematics

Final is comprehensive

Overview. Categories and corresponding methods.

1. 1st order linear, 1 equation. Integrating factor.

$y' + p(x)y = q(x)$. int. factor $u(x) = e^{(\text{integral of } p(x))}$. Multiple the eq with $u(x)$ so that the LHS becomes a ``perfect derivative''
 $(yu)' = qu$.

2. 1st order, nonlinear, separable equation. Separation of variables (the only nonlinear method we learned to fully solve equations. But Jacobian matrix can be used to study stability of nonlinear system.)
3. 2nd order, linear, constant coefficient, homogeneous. For example, $ax''+bx'+cx=0$ where a,b,c are constants. We solve for r in the char. Eq $ar^2+br+c=0$.
4. 2nd order, linear, constant coefficient with nonhomogeneous terms. For example, $ax''+bx'+cx=f(t)$ where a,b,c are constants. **General solution=a particular solution (found by ``guessing'' if $f(t)$ is of certain forms) + complementary solution from item 3.**
5. Higher order, linear, constant coefficient. Method of char. eq.
6. Systems of linear equations, homogeneous. $x'=Ax$ where x is an unknown vector and A is a constant matrix. Solve for eigenvalues in $\det(A-\lambda I)=0$ and associated eigenvectors.
7. Systems of linear equations, **nonhomogeneous**. $x'=Ax+f(t)$ where $f(t)$ is a vector-valued function. Two ways: 1) general solution = a ``guessed'' particular solution + complementary solution from item 6; 2) construct a **fundamental matrix $M(t)$** first ($\det M(t) \neq 0$), and use variation of parameter $x(t)=M(t)c(t)$, plug it into the original system and establish a new system in the form: $c'(t)=M^{-1}f(t)$
8. Nonlinear systems. 1. Look for a function $H(x,y)$ so that the solution satisfies $H(x(t), y(t))= \text{constant}$; 2. More generally, find all equilibria, linearize the system around each equilibrium by calculating the **Jacobinan matrix**.

Other things in each chapter.

Ch 1. 1st equation. Modeling of population dynamics ($dP/dt = \text{birth rate} - \text{death rate}$). Slope fields for $dy/dx = f(y)$, a so-called autonomous equation. Each solution curve $y = y(x)$ is tangent to the slopes that it passed through. Slope field can help study qualitative behavior of solutions without fully solving the equation. (use softwares on internet or `dfield7.m` in Matlab)

Ch2. Modeling. Population models. Velocity-acceleration models (if unknown is displacement, then establish a 2nd order equation)

Ch3. Mechanical vibrations. (use 2nd order equation to model a spring-mass system). Undamped, damped, **under damped**, **over damped**: physical meaning, mathematical meanings in the equations and also in the solutions.

Ch4,5. No repeated eigenvalues □ Fundamental matrix and its applications! **Imaginary eigenvalues and eigenvectors.**

Ch6. The concept and plotting of **phase portraits**. Stability, classification of equilibria (stable/unstable, sink/source, node/spiral/saddle, **check eigenvalues of the Jacobian**)

Ch7. Laplace transform (HW10, prob 3).