1. (1 pt) It can be helpful to classify a differential equation, so that we can predict the techniques that might help us to find a function which solves the equation. Two classifications are the **order of the equation** – (what is the highest number of derivatives involved) and whether or not the equation is **linear** .

Linearity is important because the structure of the the family of solutions to a linear equation is fairly simple. Linear equations can usually be solved completely and explicitly.

Determine whether or not each equation is linear:

[?] 1. $\frac{d^4y}{dt^4}+\frac{d^3y}{dt^3}+\frac{d^2y}{dt^2}+\frac{dy}{dt}=1$

[?] 2. $y''-y+y^2=0$

[?] 3. $\frac{d^3y}{dt^3}+t\frac{dy}{dt}+(cos^2(t))y=t^3$

[?] 4. $(1+y^2)\frac{d^2y}{dt^2}+t\frac{dy}{dt}+y=e^t$

Correct Answers:

- `4Linear`
- `2Nonlinear`
- `3linear`
- `2Nonlinear`

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2. (1 pt) Find the general solution to the homogeneous differential equation

$$\frac{d^2y}{dx^2}-0\frac{dy}{dx}-1y=0$$

The solution has the form

$$y=C_1f_1(x)+C_2f_2(x)$$

with

$f_1(x) =$ __________
$f_2(x) =$ __________

Left to your own devices, you will probably write down the correct answers, but in case you want to quibble, enter your answers so that f_1, f_2 are normalized with their value at $x=0$ equal to 1.

Correct Answers:

- `exp(-1*x)`
- `exp(1*x)`

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3. (1 pt)

Find the general solution to the homogeneous differential equation

$$\frac{d^2y}{dt^2}-15\frac{dy}{dt}+50y=0$$

The solution can be written in the form

$$y=C_1e^{r_1t}+C_2e^{r_2t}$$

with

$$r_1<r_2$$

Using this form, $r_1 =$ ________ and $r_2 =$ ________

Correct Answers:

- `5`
- `10`

4. (1 pt)
Find the general solution to the homogeneous differential equation

$$\frac{d^2y}{dt^2} - 9\frac{dy}{dt} = 0$$

The solution has the form

$$y = C_1 f_1(t) + C_2 f_2(t)$$

with $f_1(t) =$ ____________ and $f_2(t) =$ ____________
Left to your own devices, you will probably write down the correct answers, but in case you want to quibble, enter your answers so that f_1, f_2 are normalized with their value at $t = 0$ equal to 1.

Correct Answers:

- `exp(9*t)`
- `exp(0*t)`

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5. (1 pt)
Find the solution to the boundary value problem:

$$\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 5y = 0, \quad y(0) = 5, y(1) = 7$$

The solution is ____________________________

Correct Answers:

- `5.04524118668836*exp(1*t) + -0.0452411866883553*exp(5*t)`

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6. (1 pt)
Find the general solution to the homogeneous differential equation

$$\frac{d^2y}{dx^2} + 15\frac{dy}{dx} + 50y = 0$$

The solution has the form

$$y = C_1 f_1(x) + C_2 f_2(x)$$

with $f_1(x) =$ ____________ and $f_2(x) =$ ____________

Left to your own devices, you will probably write down the correct answers, but in case you want to quibble, enter your answers so that f_1, f_2 are normalized with their value at $x = 0$ equal to 1.

Correct Answers:

- `exp(-5*x)`
- `exp(-10*x)`

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7. (1 pt) Find y as a function of t if

$$4y'' - 9y = 0,$$

$y(0) = 8$, and $y'(0) = 8$.
$y(t) =$ ______________________________________

Correct Answers:

- `(4 - 8/3) *exp((0 - 3/2)*t) + (4 + 8/3) *exp((0 + 3/2)*t)`

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8. (1 pt) Find y as a function of t if

$$y'' + y' - 12y = 0,$$

$y(0) = 8, \quad y(1) = 7$.
$y(t) =$ ______________________________________
Remark: The initial conditions involve values at two points.

Correct Answers:

- `(8*exp(-4)-7)/(exp(-4)-exp(3))*exp(3*t)+ (-8*exp(3)+7)/(exp(-4)-exp(3))*exp(-4*t)`

9. (1 pt) Find y as a function of t if

$$2500y'' - 729y = 0$$

with $y(0) = 9, \quad y'(0) = 4.$

$y =$ ______________________

Correct Answers:

- `(9/2 - 100/27) *exp((0 - 27/50)*t) + (9/2 + 100/27) *exp((0 + 27/50)*t)`

10. (1 pt) Determine whether the following pairs of functions are linearly independent or not.

[?] 1. The Wronskian of two functions is $W(t) = t$ are the functions linearly independent or dependent?

[?] 2. $f(x) = e^{15x}$ and $g(x) = e^{15(x-1)}$

[?] 3. $f(t) = t^2 + 15t$ and $g(t) = t^2 - 15t$

Correct Answers:

- `Linearly independent`
- `Linearly dependent`
- `Linearly independent`