

1. (1 pt) Write the given second order equation as its equivalent system of first order equations.

$$u'' + 3u' + 5u = 0$$

Use  $v$  to represent the "velocity function", i.e.  $v = u'(t)$ .

Use  $v$  and  $u$  for the two functions, rather than  $u(t)$  and  $v(t)$ . (The latter confuses webwork. Functions like  $\sin(t)$  are ok.)

$$u' = \underline{\hspace{2cm}}$$

$$v' = \underline{\hspace{2cm}}$$

Now write the system using matrices:

$$\frac{d}{dt} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \underline{\hspace{1cm}} & \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} & \underline{\hspace{1cm}} \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}.$$

Correct Answers:

- $v$
- $-3v - 5u$
- $0$
- $1$
- $-5$
- $-3$

2. (1 pt) Write the given second order equation as its equivalent system of first order equations.

$$u'' + 3.5u' + 2u = 3.5 \sin(3t), \quad u(1) = -0.5, \quad u'(1) = 1$$

Use  $v$  to represent the "velocity function", i.e.  $v = u'(t)$ .

Use  $v$  and  $u$  for the two functions, rather than  $u(t)$  and  $v(t)$ . (The latter confuses webwork. Functions like  $\sin(t)$  are ok.)

$$u' = \underline{\hspace{2cm}}$$

$$v' = \underline{\hspace{2cm}}$$

Now write the system using matrices:

$$\frac{d}{dt} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \underline{\hspace{1cm}} & \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} & \underline{\hspace{1cm}} \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} \end{bmatrix}$$

and the initial value for the vector valued function is:

$$\begin{bmatrix} u(1) \\ v(1) \end{bmatrix} = \begin{bmatrix} \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} \end{bmatrix}.$$

Correct Answers:

- $v$
- $-3.5v - 2u + 3.5 \sin(3t)$
- $0$
- $1$
- $-2$
- $-3.5$
- $0$
- $3.5 \sin(3t)$
- $-0.5$
- $1$

3. (1 pt) Write the given second order equation as its equivalent system of first order equations.

$$t^2 u'' - 2.5tu' + (t^2 - 3)u = -2.5 \sin(3t)$$

Use  $v$  to represent the "velocity function", i.e.  $v = u'(t)$ .

Use  $v$  and  $u$  for the two functions, rather than  $u(t)$  and  $v(t)$ . (The latter confuses webwork. Functions like  $\sin(t)$  are ok.)

$$u' = \underline{\hspace{2cm}}$$

$$v' = \underline{\hspace{2cm}}$$

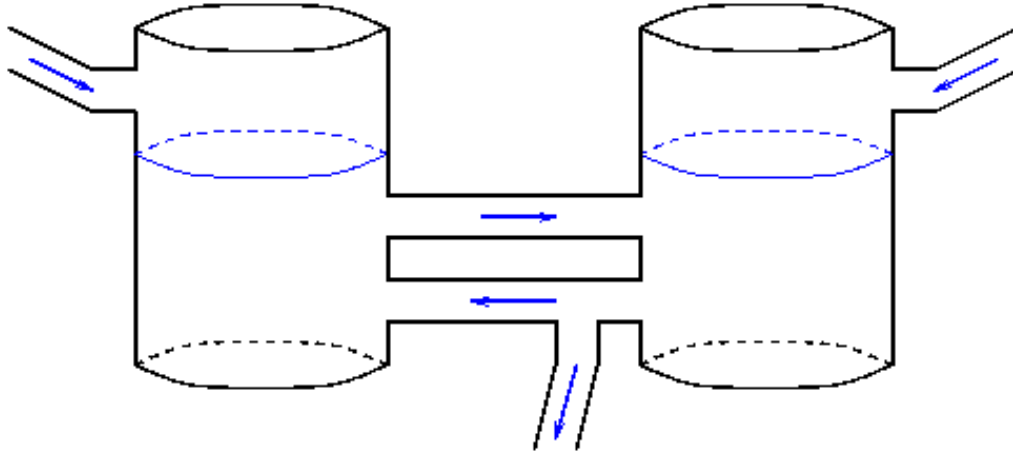
Now write the system using matrices:

$$\frac{d}{dt} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \underline{\hspace{1cm}} & \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} & \underline{\hspace{1cm}} \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} \end{bmatrix}.$$

Correct Answers:

- $v$
- $+ 2.5(1/t)v - (t^2-3)(1/t^2)u + (1/t^2)*(-2.5 \sin(3t))$
- $0$

- 1
- $-(t^2-3)(1/t^2)$
- $-2.5(1/t)$
- 0
- $(1/t^2)*(-2.5\sin(3t))$



4. (1 pt)

Consider two interconnected tanks as shown in the figure above. Tank 1 initially contains 40 L (liters) of water and 170 g of salt, while tank 2 initially contains 60 L of water and 280 g of salt. Water containing 50 g/L of salt is poured into tank 1 at a rate of 4 L/min while the mixture flowing into tank 2 contains a salt concentration of 20 g/L of salt and is flowing at the rate of 2 L/min. The two connecting tubes have a flow rate of 5.5 L/min from tank 1 to tank 2; and of 1.5 L/min from tank 2 back to tank 1. Tank 2 is drained at the rate of 6 L/min.

You may assume that the solutions in each tank are thoroughly mixed so that the concentration of the mixture leaving any tank along any of the tubes has the same concentration of salt as the tank as a whole. (This is not completely realistic, but as in real physics, we are going to work with the approximate, rather than exact description. The 'real' equations of physics are often too complicated to even write down precisely, much less solve.)

How does the water in each tank change over time?

Let  $p(t)$  and  $q(t)$  be the amount of salt in g at time  $t$  in tanks 1 and 2 respectively. Write differential equations for  $p$  and  $q$ . (As usual, use the symbols  $p$  and  $q$  rather than  $p(t)$  and  $q(t)$ .)

$p' =$

$q' =$

Give the initial values:

$$\begin{bmatrix} p(0) \\ q(0) \end{bmatrix} = \begin{bmatrix} \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} \end{bmatrix}.$$

Show the equation that needs to be solved to find a constant solution to the differential equation:

$$\begin{bmatrix} \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} \end{bmatrix} = \begin{bmatrix} \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix}.$$

A constant solution is obtained if  $p(t) = \underline{\hspace{2cm}}$  for all time  $t$  and  $q(t) = \underline{\hspace{2cm}}$  for all time  $t$ .

Correct Answers:

- $50 \cdot 4 - (5.5/40)p + (1.5/60)q$
- $20 \cdot 2 - (1.5/60 + 6/60)q + (5.5/40)p$
- 170
- 280
- -200
- -40

- $-0.1375$
- $0.025$
- $0.1375$
- $-0.125$
- $1890.90909090909$
- $2400$

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1. (1 pt) Calculate the eigenvalues of this matrix:

If you select the "integral curves utility" from the main menu, will also be able to plot the integral curves of the associated differential equations. ]

$$A = \begin{bmatrix} 3 & -105 \\ -84 & 24 \end{bmatrix}$$

smaller eigenvalue = \_\_\_\_\_

associated eigenvector = ( \_\_\_\_\_ , \_\_\_\_\_ )

larger eigenvalue = \_\_\_\_\_

associated, eigenvector = ( \_\_\_\_\_ , \_\_\_\_\_ )

---

If  $y' = Ay$  is a differential equation, how would the solution curves behave?

- A. All of the solution curves would run away from 0. (Unstable node)
  - B. All of the solutions curves would converge towards 0. (Stable node)
  - C. The solution curves converge to different points.
  - D. The solution curves would race towards zero and then veer away towards infinity. (Saddle)
- 

*Correct Answers:*

- -81
  - ( 5, 4 )
  - 108
  - ( -3, 3 )
  - D
- 

2. (1 pt) Calculate the eigenvalues of this matrix:

If you select the "integral curves utility" from the main menu, will also be able to plot the integral curves of the associated differential equations. ]

$$A = \begin{bmatrix} 7 & 3 \\ -0.9999999999999999 & 11 \end{bmatrix}$$

smaller eigenvalue = \_\_\_\_\_

associated eigenvector = ( \_\_\_\_\_ , \_\_\_\_\_ )

larger eigenvalue = \_\_\_\_\_

associated, eigenvector = ( \_\_\_\_\_ , \_\_\_\_\_ )

---

If  $y' = Ay$  is a differential equation, how would the solution curves behave?

- A. The solution curves would race towards zero and then veer away towards infinity. (Saddle)
  - B. All of the solution curves would run away from 0. (Unstable node)
  - C. The solution curves converge to different points.
  - D. All of the solutions curves would converge towards 0. (Stable node)
- 

*Correct Answers:*

- The eigenvalues and eigenvectors are {10,8} and the eigenvectors are ( 1 ,1 ) and ( 3 ,1 )
  - The eigenvalues and eigenvectors are {10,8} and the eigenvectors are ( 1 ,1 ) and ( 3 ,1 )
  - B
- 

3. (1 pt) Match the differential equations and their vector valued function solutions:

It will be good practice to multiply at least one solution out fully, to make sure that you know how to do it, but you can get the other answers quickly by process of elimination and just multiply out one row element.

\_\_\_1.  $y'(t) = \begin{bmatrix} 15 & 0 & 0 \\ 4 & 20 & -15 \\ 4 & 30 & -25 \end{bmatrix} y(t)$

—2.  $y'(t) = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix} y(t)$

—3.  $y'(t) = \begin{bmatrix} -86 & 218 & -160 \\ 73 & -49 & 80 \\ 111 & -138 & 165 \end{bmatrix} y(t)$

A.

$$y(t) = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} e^{5t}$$

B.

$$y(t) = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix} e^{-2t}$$

C.

$$y(t) = \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix} e^{45t}$$

Correct Answers:

- A
- B
- C

4. (1 pt) Solve the system

$$\frac{dx}{dt} = \begin{bmatrix} -10 & 8 \\ -12 & 10 \end{bmatrix} x$$

with the initial value  $x(0) = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$ .

$$x(t) = \begin{bmatrix} \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} \end{bmatrix}.$$

Correct Answers:

- $-1 \cdot e^{(-2 \cdot t)} \cdot 1 + 2 \cdot e^{(2 \cdot t)} \cdot 2$
- $-1 \cdot e^{(-2 \cdot t)} \cdot 1 + 2 \cdot e^{(2 \cdot t)} \cdot 3$

5. (1 pt) Solve the system

$$\frac{dx}{dt} = \begin{bmatrix} 16 & 16 \\ 12 & 12 \end{bmatrix} x$$

with the initial value  $x(0) = \begin{bmatrix} -15 \\ -20 \end{bmatrix}$ .

$$x(t) = \begin{bmatrix} \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} \end{bmatrix}.$$

Correct Answers:

- $-5 \cdot e^{(28 \cdot t)} \cdot 4 + -5 \cdot -1$
- $-5 \cdot e^{(28 \cdot t)} \cdot 3 + -5 \cdot 1$

6. (1 pt) Multiplying the differential equation

$$\frac{df}{dt} + af(t) = g(t),$$

where  $a$  is a constant and  $g(t)$  is a smooth function, by  $e^{at}$ , gives

$$e^{at} \frac{df}{dt} + e^{at} af(t) = e^{at} g(t),$$

$$\frac{d}{dt} (e^{at} f(t)) = e^{at} g(t),$$

$$e^{at} f(t) = \int e^{at} g(t) dt,$$

$$f(t) = e^{-at} \int e^{at} g(t) dt.$$

Use this to solve the initial value problem

$$\frac{dx}{dt} = \begin{bmatrix} 1 & -3 \\ 0 & 2 \end{bmatrix} x,$$

$$\text{with } x(0) = \begin{bmatrix} 3 \\ -2 \end{bmatrix},$$

i.e. find first  $x_2(t)$  and then  $x_1(t)$ .

$$x_1(t) = \underline{\hspace{2cm}},$$

$$x_2(t) = \underline{\hspace{2cm}}.$$

*Correct Answers:*

- $-3 \cdot -2 / (2 - 1) \cdot e^{(2 \cdot t)} + (3 - -3 \cdot -2 / (2 - 1)) \cdot e^{(1 \cdot t)}$
- $-2 \cdot e^{(2 \cdot t)}$

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1. (1 pt) Consider the system of differential equations

$$\frac{dx}{dt} = -1.4x + 0.75y,$$

$$\frac{dy}{dt} = 1.66666666666667x - 3.4y.$$

For this system, the smaller eigenvalue is \_\_\_\_\_ and the larger eigenvalue is \_\_\_\_\_.

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If  $y' = Ay$  is a differential equation, how would the solution curves behave?

- A. All of the solution curves would run away from 0. (Unstable node)
- B. The solution curves converge to different points.
- C. The solution curves would race towards zero and then veer away towards infinity. (Saddle)
- D. All of the solutions curves would converge towards 0. (Stable node)

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The solution to the above differential equation with initial values  $x(0) = 2$ ,  $y(0) = 6$  is

$$x(t) = \underline{\hspace{2cm}},$$

$$y(t) = \underline{\hspace{2cm}}.$$

Correct Answers:

- -3.9
- -0.9
- D
- $-1.55555555555556*0.75*\exp(-3.9*t)+4.22222222222222*0.75*\exp(-0.9*t)$
- $-1.55555555555556*(-3.9 + 1.4)*\exp(-3.9*t)+4.22222222222222*(-0.9 + 1.4)*\exp(-0.9*t)$

---

2. (1 pt) Consider the interaction of two species of animals in a habitat. We are told that the change of the populations  $x(t)$  and  $y(t)$  can be modeled by the equations

$$\frac{dx}{dt} = 0.4x + 1.5y,$$

$$\frac{dy}{dt} = 0.5x - 0.6y.$$

For this system, the smaller eigenvalue is \_\_\_\_\_ and the larger eigenvalue is \_\_\_\_\_.

1. What kind of interaction do we observe?

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If  $y' = Ay$  is a differential equation, how would the solution curves behave?

- A. All of the solution curves would run away from 0. (Unstable node)
- B. The solution curves converge to different points.
- C. All of the solutions curves would converge towards 0. (Stable node)
- D. The solution curves would race towards zero and then veer away towards infinity. (Saddle)

---

The solution to the above differential equation with initial values  $x(0) = 6$ ,  $y(0) = 2$  is

$$x(t) = \underline{\hspace{2cm}},$$

$$y(t) = \underline{\hspace{2cm}}.$$

Correct Answers:

- -1.1
- 0.9
- Symbiosis
- D
- $-0*1.5*\exp(-1.1*t)+4*1.5*\exp(0.9*t)$
- $-0*(-1.1-0.4)*\exp(-1.1*t)+4*(0.9-0.4)*\exp(0.9*t)$

3. (1 pt) Consider the interaction of two species of animals in a habitat. We are told that the change of the populations  $x(t)$  and  $y(t)$  can be modeled by the equations

$$\frac{dx}{dt} = 0.5x - 0.8y,$$

$$\frac{dy}{dt} = -0.2x + 1.1y.$$

For this system, the smaller eigenvalue is \_\_\_\_\_ and the larger eigenvalue is \_\_\_\_\_.

1. What kind of interaction do we observe?

If  $y' = Ay$  is a differential equation, how would the solution curves behave?

- A. All of the solution curves would run away from 0. (Unstable node)
- B. The solution curves would race towards zero and then veer away towards infinity. (Saddle)
- C. All of the solutions curves would converge towards 0. (Stable node)
- D. The solution curves converge to different points.

The solution to the above differential equation with initial values  $x(0) = 9$ ,  $y(0) = 4$  is

$$x(t) = \underline{\hspace{2cm}},$$

$$y(t) = \underline{\hspace{2cm}}.$$

Correct Answers:

- 0.3
- 1.3
- Competition
- A
- $-13*0.8*\exp(0.3*t)+1.75*0.8*\exp(1.3*t)$
- $-13*(0.3-0.5)*\exp(0.3*t)+1.75*(1.3-0.5)*\exp(1.3*t)$

4. (1 pt) Consider the interaction of two species of animals in a habitat. We are told that the change of the populations  $x(t)$  and  $y(t)$  can be modeled by the equations

$$\frac{dx}{dt} = 1.4x + 1y,$$

$$\frac{dy}{dt} = 1.25x - 0.6y.$$

For this system, the smaller eigenvalue is \_\_\_\_\_ and the larger eigenvalue is \_\_\_\_\_.

1. What kind of interaction do we observe?

If  $y' = Ay$  is a differential equation, how would the solution curves behave?

- A. The solution curves converge to different points.
- B. All of the solution curves would run away from 0. (Unstable node)
- C. All of the solutions curves would converge towards 0. (Stable node)
- D. The solution curves would race towards zero and then veer away towards infinity. (Saddle)

The solution to the above differential equation with initial values  $x(0) = 6$ ,  $y(0) = 3$  is

$$x(t) = \underline{\hspace{2cm}},$$

$$y(t) = \underline{\hspace{2cm}}.$$

Correct Answers:

- -1.1
- 1.9
- Symbiosis
- D
- $-0*1*\exp(-1.1*t)+6*1*\exp(1.9*t)$



$$\bullet -0 * (-1.1 - 1.4) * \exp(-1.1 * t) + 6 * (1.9 - 1.4) * \exp(1.9 * t)$$

5. (1 pt) Consider the interaction of two species of animals in a habitat. We are told that the change of the populations  $x(t)$  and  $y(t)$  can be modeled by the equations

$$\frac{dx}{dt} = 1.6x - 0.5y,$$

$$\frac{dy}{dt} = -2.5x + 3.6y.$$

For this system, the smaller eigenvalue is \_\_\_\_\_ and the larger eigenvalue is \_\_\_\_\_.

1. What kind of interaction do we observe?

If  $y' = Ay$  is a differential equation, how would the solution curves behave?

- A. All of the solution curves would run away from 0. (Unstable node)
- B. All of the solutions curves would converge towards 0. (Stable node)
- C. The solution curves converge to different points.
- D. The solution curves would race towards zero and then veer away towards infinity. (Saddle)

The solution to the above differential equation with initial values  $x(0) = 9$ ,  $y(0) = 5$  is

$$x(t) = \underline{\hspace{2cm}},$$

$$y(t) = \underline{\hspace{2cm}}.$$

Correct Answers:

- 1.1
- 4.1
- Competition
- A
- $-16.6666666666667 * -0.5 * \exp(1.1 * t) - 1.3333333333333 * -0.5 * \exp(4.1 * t)$
- $-16.6666666666667 * (1.1 - 1.6) * \exp(1.1 * t) - 1.3333333333333 * (4.1 - 1.6) * \exp(4.1 * t)$

---

1. (1 pt) Solve the system

$$\frac{dx}{dt} = \begin{bmatrix} 6 & 3 \\ -15 & -6 \end{bmatrix} x$$

$$\text{with } x(0) = \begin{bmatrix} 3 \\ 4 \end{bmatrix}.$$

Give your solution in real form.

$$x_1 = \underline{\hspace{2cm}},$$

$$x_2 = \underline{\hspace{2cm}}.$$

---

1. Describe the trajectory.

1. What kind of interaction do we observe?

*Correct Answers:*

- $3 * (-(-2/1) * \sin(3*t) + \cos(3*t)) + (4/1) * (\sin(3*t))$
  - $3 * (-(-2*-2/1) * \sin(3*t) - 1 * \sin(3*t)) + 4 * (\cos(3*t) + (-2/1) * \sin(3*t))$
  - Ellipse clockwise
  - Predator-prey
- 

2. (1 pt) Solve the system

$$\frac{dx}{dt} = \begin{bmatrix} -5 & -3 \\ 3 & -5 \end{bmatrix} x$$

$$\text{with } x(0) = \begin{bmatrix} 8 \\ 5 \end{bmatrix}.$$

Give your solution in real form.

$$x_1 = \underline{\hspace{2cm}},$$

$$x_2 = \underline{\hspace{2cm}}.$$

---

1. Describe the trajectory.

1. What kind of interaction do we observe?

*Correct Answers:*

- $e^{(-5*t)} * (8 * \cos(3*t) - 5 * \sin(3*t))$
  - $e^{(-5*t)} * (8 * \sin(3*t) + 5 * \cos(3*t))$
  - Spiral inward counterclockwise
  - Predator-prey
- 

3. (1 pt) Solve the system

$$\frac{dx}{dt} = \begin{bmatrix} 3 & -3 \\ 6 & -3 \end{bmatrix} x$$

$$\text{with } x(0) = \begin{bmatrix} 6 \\ 2 \end{bmatrix}.$$

Give your solution in real form.

$$x_1 = \underline{\hspace{2cm}},$$

$$x_2 = \underline{\hspace{2cm}}.$$

---

1. Describe the trajectory.

1. What kind of interaction do we observe?

*Correct Answers:*

- $6 * (- (1/-1) * \sin(3*t) + \cos(3*t)) + (2/-1) * (\sin(3*t))$
  - $6 * (- (1*1/-1) * \sin(3*t) + 1 * \sin(3*t)) + 2 * (\cos(3*t) + (1/-1) * \sin(3*t))$
  - Ellipse counterclockwise
  - Predator-prey
-

---

4. (1 pt) Solve the system

$$\frac{dx}{dt} = \begin{bmatrix} -0.5 & -3 \\ 3 & -0.5 \end{bmatrix} x$$

$$\text{with } x(0) = \begin{bmatrix} 4 \\ 9 \end{bmatrix}.$$

Give your solution in real form.

$$x_1 = \underline{\hspace{2cm}},$$

$$x_2 = \underline{\hspace{2cm}}.$$

---

? 1. Describe the trajectory.

? 1. What kind of interaction do we observe?

*Correct Answers:*

- $e^{-0.5t} * (4\cos(3t) - 9\sin(3t))$
  - $e^{-0.5t} * (4\sin(3t) + 9\cos(3t))$
  - Spiral inward counterclockwise
  - Predator-prey
- 

5. (1 pt) Solve the system

$$\frac{dx}{dt} = \begin{bmatrix} 3 & -2 \\ 2 & 3 \end{bmatrix} x$$

$$\text{with } x(0) = \begin{bmatrix} 7 \\ 5 \end{bmatrix}.$$

Give your solution in real form.

$$x_1 = \underline{\hspace{2cm}},$$

$$x_2 = \underline{\hspace{2cm}}.$$

---

? 1. Describe the trajectory.

? 1. What kind of interaction do we observe?

*Correct Answers:*

- $e^{3t} * (7\cos(2t) - 5\sin(2t))$
- $e^{3t} * (7\sin(2t) + 5\cos(2t))$
- Spiral outward counterclockwise
- Predator-prey

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1. (1 pt) Solve the system

$$\frac{dx}{dt} = \begin{bmatrix} 1 & -4 \\ 4 & -7 \end{bmatrix} x$$

$$\text{with } x(0) = \begin{bmatrix} 3 \\ 2 \end{bmatrix}.$$

Give your solution in real form.

$$x_1 = \underline{\hspace{2cm}},$$

$$x_2 = \underline{\hspace{2cm}}.$$

---

If  $y' = Ay$  is a differential equation, how would the solution curves behave?

- A. All of the solution curves would run away from 0. (Unstable node)
  - B. The solution curves would race towards zero and then veer away towards infinity. (Saddle)
  - C. The solution curves converge to different points.
  - D. All of the solutions curves would converge towards 0. (Stable node)
- 

*Correct Answers:*

- $e^{(-3*t)} * (3+4*t)$
  - $e^{(-3*t)} * (2+4*t)$
  - D
- 

2. (1 pt) Solve the system

$$\frac{dx}{dt} = \begin{bmatrix} 2 & 1.5 \\ -1.5 & -1 \end{bmatrix} x$$

$$\text{with } x(0) = \begin{bmatrix} 3 \\ -2 \end{bmatrix}.$$

Give your solution in real form.

$$x_1 = \underline{\hspace{2cm}},$$

$$x_2 = \underline{\hspace{2cm}}.$$

---

If  $y' = Ay$  is a differential equation, how would the solution curves behave?

- A. All of the solution curves would run away from 0. (Unstable node)
  - B. The solution curves would race towards zero and then veer away towards infinity. (Saddle)
  - C. All of the solutions curves would converge towards 0. (Stable node)
  - D. The solution curves converge to different points.
- 

*Correct Answers:*

- $\exp(t/2) * (3+1.5*t)$
  - $-\exp(t/2) * (2+1.5*t)$
  - A
- 

3. (1 pt) Solve the system

$$\frac{dx}{dt} = \begin{bmatrix} -2.5 & 1.5 \\ -1.5 & 0.5 \end{bmatrix} x$$

$$\text{with } x(0) = \begin{bmatrix} 3 \\ -1 \end{bmatrix}.$$

Give your solution in real form.

$$x_1 = \underline{\hspace{2cm}},$$

$$x_2 = \underline{\hspace{2cm}}.$$

---

If  $y' = Ay$  is a differential equation, how would the solution curves behave?

- A. The solution curves converge to different points.
- B. The solution curves would race towards zero and then veer away towards infinity. (Saddle)
- C. All of the solution curves would run away from 0. (Unstable node)
- D. All of the solutions curves would converge towards 0. (Stable node)

---

*Correct Answers:*

- $\exp(-t) * (3-6*t)$
- $-\exp(-t) * (1+6*t)$
- D

---

**4.** (1 pt) Solve the system

$$\frac{dx}{dt} = \begin{bmatrix} 3 & 9 \\ -1 & -3 \end{bmatrix} x$$

with  $x(0) = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$ .

Give your solution in real form.

$x_1 =$  \_\_\_\_\_,

$x_2 =$  \_\_\_\_\_.

*Correct Answers:*

- $2+42*t$
- $4-14*t$

---

1. (1 pt) Find the equilibrium solution for

$$x_1'(t) = -7.2 + 1.2x_1 - 0.8x_2$$

$$x_2'(t) = -13.8 + 2.1x_1 - 1.2x_2$$

$$x_1(0) = 12; x_2(0) = 8$$

Equilibrium:  $x_1^e =$  \_\_\_\_\_,  
 $x_2^e =$  \_\_\_\_\_.

---

1. Describe the trajectory.

1. What kind of interaction do we observe?

*Correct Answers:*

- 10
  - 6
  - Ellipse counterclockwise
  - Predator-prey
- 

2. (1 pt) Find the equilibrium solution for

$$x_1'(t) = -6.2 + 1.1x_1 - 0.8x_2$$

$$x_2'(t) = -13.8 + 2.1x_1 - 1.2x_2$$

$$x_1(0) = 11; x_2(0) = 4$$

Equilibrium:  $x_1^e =$  \_\_\_\_\_,  
 $x_2^e =$  \_\_\_\_\_.

---

1. Describe the trajectory.

1. What kind of interaction do we observe?

*Correct Answers:*

- 10
- 6
- Spiral inward counterclockwise
- Predator-prey

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3. (1 pt) Find the equilibrium solution for

$$x_1'(t) = -9.7 + 1.2x_1 - 0.5x_2$$

$$x_2'(t) = -9.8 + 1.4x_1 - 0.8x_2$$

$$x_1(0) = 20; x_2(0) = 34$$

Equilibrium:  $x_1^e =$  \_\_\_\_\_,  
 $x_2^e =$  \_\_\_\_\_.

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If  $y' = Ay$  is a differential equation, how would the solution curves behave?

- A. The solution curves converge to different points.
- B. All of the solutions curves would converge towards the equilibrium point. (Stable node)
- C. The solution curves would race towards the equilibrium point and then veer away towards infinity. (Saddle)
- D. All of the solution curves would run away from the equilibrium point. (Unstable node)

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*Correct Answers:*

- 11
- 7
- C