MAT 274 HW 6

Due Friday Oct 29 in class.
(15'+20'+20'+15')

1. Find the general solution to

$$x''(t) + 2x'(t) + 3x(t) + 5\sin 2t = -12e^{-3t}$$

and show that any solution $x(t)$ to this DE satisfies the following asymptotic behavior

$$\lim_{t\to\infty}(x(t) - \sin 2t - 2\cos 2t) = 0.$$

2. Consider the system

$$(*) \qquad \frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

from the previous problem.

a) Find two linearly independent solutions that are also **REAL**;

b) Use the Wronskian to prove linear independency;

c) Draw a phase portrait **by hand** and justify your choice of source/sink, node/spiral/saddle point. In the case of spiral, also justify the clockwise/counterclockwise direction of rolling.

3. Consider the same system (*) as in the previous problem.

a) Find the particular solution satifying initial condition $x(0) = -2$, $y(0) = 1$;

b) Show that, for this particular solution,

$$2x^2(t) + y^2(t) + 2x(t)y(t) = 5e^{6t}$$

and therefore

$$(**) \qquad \frac{d}{dt}[2x^2(t) + y^2(t) + 2x(t)y(t)] = 6[2x^2(t) + y^2(t) + 2x(t)y(t)]$$

c) Prove that (**) is true for ANY solution to (*).

4. With the help of Problem 2, find the general solution to

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} 6e^{-t} \\ 4e^{-t} \end{pmatrix}$$

You may start with finding the particular solution.