MAT274 HW5

DUE IN CLASS, OCTOBER 13, 2010.
READINGS: §4.1, 5.1, 5.2 (TILL EXAMPLE 2) OF EDWARDS & PENNEY

1. Consider a spring-mass system with spring constant 5.
   (a) Establish a second order DE in terms of the unknown displacement $x(t)$.
   (b) Now, consider two unknown functions: displacement $x(t)$ and velocity $y(t)$. Establish a **system** of 2 equations with **first** order derives. One DE should come from Newton's second law and the other from the definition of velocity.
   (c) Solve the second DE in part (a) with initial conditions $x(0) = 2$ and $x'(0) = -10$. Then, use it to construct the particular solution for the system established in part (b) with initial conditions $x(0) = 2$ and $y(0) = -10$.
2. (10 pt) (Show all your work) For the following matrix A, find its (a) determinant; (b) inverse; (c) eigenvalues; (d) eigenvectors.

$$A = \begin{pmatrix} -3 & 4 \\ 6 & -5 \end{pmatrix}.$$

3. With results from Problem 1, solve for the particular solution to the following DE (all coefficients in your final answer should be integers)

$$\frac{dx}{dt} = -3x + 4y, \quad x(0) = -1$$
$$\frac{dy}{dt} = 6x - 5y, \quad y(0) = 4.$$

We discussed the mathematical and physical meanings of curvature $\kappa(t)$ for parameterized curve

$$x = x(t), \quad y = y(t), \quad z = z(t).$$

For a circle, it is one over the radius (and therefore $\kappa(t)$ has unit 1/length). For a arbitrary curve, it is one over the radius of the osculating circle at a given point. In class, we learned that curvature is the magnitude of