MAT274 HW4

DUE IN CLASS, OCTOBER 4, 2010.
READINGS: §3.3 — 3.5 OF EDWARDS & PENNEY

Points: 20'+50'+50'

1. **Show all your work.** Find the function y_1 of t which is the solution of

$$16y'' - 64y = 0$$

with initial conditions $y_1(0) = 1, \quad y_1'(0) = 0.$
$y_1 =$ __________________________

Find the function y_2 of t which is the solution of

$$16y'' - 64y = 0$$

with initial conditions $y_2(0) = 0, \quad y_2'(0) = 1.$
$y_2 =$ __________________________

Find the Wronskian

$$W(t) = W(y_1, y_2).$$

$W(t) =$ __________________________

Briefly explain how the above computation suggests that y_1 and y_2 form a linearly independent set of solutions of

$$16y'' - 64y = 0.$$

2. Find general solutions to the following DEs respectively
   (2 i)

$$y''(x) + 2y'(x) + 5y(x) + 7e^{-3x} = 0.$$

(2 ii)

$$y''(t) + 4y(t) = 6e^{2t}.$$

(2 iii)

$$y''(t) + y'(t) - 6y(t) = 3e^{2t} \qquad \text{Hint: guessing } y_p = ae^{2t} \text{ won't work,}$$

because e^{2t} is already part of the **homogeneous** solution.

Instead, guess $y_p = ate^{2t}$.

(2 iv)

$\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = -12x^2$. Hint: take a guess $y_p = ax^2 + bx + c$ and solve for a, b, c.

(2 v)

$$\frac{d^2y}{dx^2} - 9\frac{dy}{dx} = 5xe^{-2x} + 2x^2. \text{ Note: to accommodate the } 5xe^{-2x} \text{ term,}$$

use $(ax + b)e^{-2x}$ as **part** of your "guess".

3. The suspension system of a car uses a spring-mass system to absorb impacts and dampers (or shock absorbers) to control spring motions. Suppose the spring follows the Hooke's law of elasticity which means the displacement of the mass is in direct proportion with the load added to it

$$F_{elastic} = -kx.$$

Let's fix $k = 25$ throughout this problem. Also fix the mass $m = 1$.

(3 i) If no damper is used in the system, what is the governing DE and what is the general solution to this DE? Here, use the time t as the independent variable and the displacement of the mass $x(t)$ as the unknown function. Then, apply Newton's second law of motion. The established DE should only involves $x(t)$ and its derivative(s).

(3 ii) With such an UNdamped suspension system in part (3i), the cat hits a bump at $t = 0$ and the suspension starts working. Assume the initial displacement of the mass is $x(0) = 0$ and initial velocity $x'(0) = -20$. Find the particular solution to the DE established in part (a) with this set of initial conditions and plot the graph of x vs t.

(3 iii) Now, introduce a damper in the suspension to add a drag force that is proportional to the velocity of the mass,

$$F_{drag} = -\gamma \frac{dx}{dt}.$$

Here, γ is a postive constant and the negative sign above indicates the drag force always works against the motion of the mass submerged in it. Fix $\gamma = 6$. Establish a new DE and find the particular solution with the same initial conditions as in part (3ii), i.e. $x(0) = 0$ and $x'(0) = -20$.

(3 iv) Plot x vs t with the particular solution from part (3iii). Estimate how long it takes for the amplitude of the oscillation to reduce by half.

(3 v) The state-of-the-art suspension system allows the driver to change the coefficient of drag force γ in part (3iii). Now, what is the critical value γ_c such that, upon gearing the constant $\gamma > \gamma_c$, the driver won't feel oscillating motion at all after hitting a bump? — the catch, however, for having overdamped suspension is that the driver experiences large shock in relative short times.