MAT274 HW2

Readings: §1.6, 2.1 – 2.3 of Edwards & Penney

[Due in class, September 8, 2010]

1. Consider the differential equation

$$\frac{dy}{dt} = y(y-4).$$

   i) Draw its slope field.

   ii) Describe the behavior of $y(t)$ as $t \to \infty$. Its asymptotic behavior may depend on the initial condition and you should specify all types (increase, decrease, constant, etc).

   iii) Find the general solution for $y(t)$. Is your result consistent with ii)?

2. It is the pollen season. Consider the air in an apartment with volume V. Let the internal concentration of pollen be $x(t)$ and the ambient (external) concentration $a(t)$. Assume, inside the apartment, pollen is evenly distributed so that the total amount of pollen is given by

$$P(t) = V \cdot x(t).$$

   Let air flow into and out of the apartment at a rate of $r(t)$.

   i) How much air flows in and out during a time period dt? How much pollen is in and out, respectively?

   ii) Find the corresponding change in the total amount of pollen $dP(t)$.

   iii) Write down a differential equation for the internal concentration $x(t)$.

   iv) Suppose $a(t), r(t)$ are known. Propose an applicable method to solve for $x(t)$. DO NOT actually solve it.

3. Find **all** equilibrium solutions for

$$\frac{dy}{dx} = y\sin(\pi y), \qquad y \in (-3/2, 3/2)$$

and classify them as asymptotically stable, unstable or semi-stable. Use slope field to support your argument.

4. Using the substitution $v = y/x$ to transform the following DE into another DE in terms of unknown function $v(x)$. Then, solve for $v(x)$ in this new DE and finally find the general solution $y(x)$ of the original equation.

$$(x - 3y)y' = (y + x).$$

5. Let an object (with mass m) start a vertical motion near the surface of the Earth with initial velocity v_0. Let the resistance be proportional to v^p, that is $F_R = -kv|v|^{p-1}$

   i) Briefly explain why we use an absolute value in the formula for F_R.

   ii) Write down an DE for the velocity v.

   iii) Without solving the DE, use a slope field to study the asymptotic behavior of $v(t)$ as $t \to \infty$. In particular, find the terminal speed. Does it matter if the initial velocity $v_0 > 0$ or $v_0 < 0$?