MAT274 Homework 1

Due: Wed 9/1

Readings: §1.1, 1.3 – 1.5 of *Edwards & Penny*

Points: 10 points each for P1–P5; 20 points each for P6–P7.

1. The equation
$$x\frac{dy}{dx} + 3y = 2x^5$$
is a ____ order differential equation that is _____ (linear/nonlinear). Verify that $y(x) = \frac{1}{4}x^5 + Cx^{-3}$ with constant C is a general solution. Then, determine the value of C using initial condition $y(2) = 1$.

2. Use the *Maple* command

```
with(DEtools):
DEplot(x*diff(y(x),x)+3*y(x)=2*x^5, x=-5...5, y=-5..5);
```

to help sketch the slop field and *several* solutions of the previous problem. Then, indicate the curve of the particular solution that satifies the given initial condition $y(2) = 3$.

3. Suppose a population P is a function of time t. If the birth rate is 2 times the square of P and the death rate is a constant 3, what is the differential equation that models the population dynamics? Is the equation linear or nonlinear? How many initial conditions is needed to determine a specific solution?

4. **Without** help of computers or calculators, sketch the slope field and *several* typical solutions of the differential obtained from previous problem. Indicate the curve that satisfes $y(0) = 1$ and the curve that satisfes $y(0) = 2$. Is the population increasing or decreasing according to these two curves? (The skill of sketching slope fields **by hand** is required in this class).

5. Give an example of a 2nd order, linear, homogeneous differential equation with constant coefficients. Specify 2 initial conditions, too.

6. Find general solutions (implicit if necessary, explicit if convenient) of the following equations

    (a) $y' = y^3/x$;

    (b) $\frac{dy}{dx} = e^{2y-3} \cos x$.

7. Find particular solutions of

    (a) $xy' - 2y = x^3$, $\qquad y(2) = 8$;

    (b) $y' = e^{(x^2)} + y + 2xy$, $\qquad y(0) = 1$.