Answers to Maths B (EE1.MAB) exam papers

Spring 2006

1. (a) $1 - \frac{1}{4}x^2 + \frac{5}{32}x^4 - \frac{15}{128}x^6 + \cdots$, (b) $e^{2x}\cos 2x = 1 + 2x - \frac{8}{3}x^3 + \cdots$.
2. (a) $\frac{1}{4}$, (b) $\frac{1}{3}$.
3. (i) (a) $\cosh x = \frac{13}{5}$, (b) $\tanh x = \frac{12}{13}$,
(c) $\sinh 2x = \frac{312}{25}$.
(ii) $\sinh^{-1}\left(\frac{x-1}{3}\right) + c$
4. (i) $y = \frac{1}{x^2 - c}$, (ii) $y = \tan\left(\frac{x^3}{3} + \frac{\pi}{4}\right)$, (iii) $y = \frac{x}{3} - \frac{1}{9} + c\,e^{-3x}$
5. (i) $y = A\,e^{-5x} + B\,e^{-2x}$, (ii) $y = e^{-x}(A\cos 2x + B\sin 2x) + \frac{1}{5}x^2 - \frac{4}{25}x - \frac{27}{125}$.
6. 1.164
7. (i) $\frac{\pi^2}{2} + 2$, (ii) $\frac{13}{8}$
8. (ii) because the function is even
(iii) $a_n = \frac{2}{n}\sin\frac{n\pi}{2}$ if $n = 1, 2, 3, \ldots$, and $a_0 = \pi$.

Spring 2007

1. (i) $1 + 3x^2 + 6x^4 + 10x^6 + \cdots$
(ii) $\ln(1+x) = x - x^2/2 + x^3/3 - \cdots$, $\ln(1+2x) = 2x - 2x^2 + \frac{8}{3}x^3 + \cdots$,
$\ln((1+2x)(1+x)) = \ln(1+2x) + \ln(1+x) = 3x - \frac{5}{2}x^2 + 3x^3 + \cdots$.
2. (i) $\frac{1}{12}$, (ii) -1, (iii) $\frac{9}{2}$.
3. (i) (a) $\frac{1}{2}(x^2 - 1/x^2)$, (b) 0
(ii) $2\sinh^{-1}(1)$ or $2\ln(1+\sqrt{2})$
4. (i) $y = Ae^{x^4/4}$, (ii) $y = \frac{2}{1+e^{-2x}}$,
(iii) $y = \frac{1}{2} + \frac{c}{(x+1)^2}$. Depending on how you do the calculations you might instead get
$y = \frac{x^2/2 + x}{(x+1)^2} + \frac{d}{(x+1)^2}$ which is equivalent.
5. (i) $y = e^{-x}(-2\cos 2x - \frac{1}{2}\sin 2x)$,
(ii) $y = Ae^{-2x} + Be^{x} - \frac{3}{20}\cos 2x + \frac{1}{20}\sin 2x$
6.

$$x_{n+1} = x_n - \frac{(x_n^6 - 2x_n^2 - 1)}{6x_n^5 - 4x_n} \qquad \text{root is } 1.272$$

7. (i) 4, (ii) $\frac{\pi}{4}(1 - e^{-1})$
8. (ii) because the function is odd
(iii) $b_n = \frac{2}{n}(-1)^{n+1}$

Spring 2008

1. (i) $1 + 8x + 40x^2 + 160x^3 + 560x^4 + \cdots$,
(ii) $x^2 - \frac{1}{6}x^6 + \frac{1}{120}x^{10} + \cdots$

$$\int_0^1 \sin(x^2)\,dx \approx 0.3103.$$

2. (i) 8, (ii) $\frac{3}{5}$, (iii) 6
3. (i) $\tanh(\ln x) = (x^2 - 1)/(x^2 + 1)$.

(ii) $\sin^2 2x = \frac{1}{2} - \frac{1}{2}\cos 4x$ so, by Osborn's rule, $\sinh^2 2x = \frac{1}{2}\cosh 4x - \frac{1}{2}$.

$$\int \sinh^2 2x \, dx = \frac{\sinh 4x}{8} - \frac{x}{2} + c$$

$$\int \frac{dx}{\sqrt{x^2 - 6x - 7}} = \cosh^{-1}\frac{x-3}{4} + c$$

4. (i) $y = \sqrt{-x^2 + d}$, (ii) $y = \left(\frac{3}{2}x^2 + 3x^3 + 216\right)^{1/3}$,
(iii) $y = -\frac{\cos 3x}{3x^2} + c/x^2$.
5. (i) $y = \frac{2}{3}e^{-5x} + \frac{1}{3}e^{4x}$, (ii) $y = e^{2x}(A\cos 3x + B\sin 3x) + \frac{3}{25}e^{-2x}$.
6. 0.5049
7. (i) 1, (ii) $\pi/4$.
8. the function is even so $b_n = 0$ for all n.

$$a_n = \frac{4(-1)^n}{n^2}, \qquad a_0 = \frac{2\pi^2}{3}.$$

Spring 2009

1. (i) $(1+3x)^{1/3} = 1 + x - x^2 + \frac{5}{3}x^3 + \cdots$
(ii) $e^{-x^2} = 1 - x^2 + \frac{1}{2}x^4 + \cdots$, $\cos 2x = 1 - 2x^2 + \frac{2}{3}x^4 + \cdots$,
$e^{-x^2}\cos 2x = 1 - 3x^2 + \frac{19}{6}x^4 + \cdots$.
2. (i) 6, (ii) $\frac{1}{2}$, (iii) $-6/\pi$.
3. (i) differentiate twice and use $\cosh^2 - \sinh^2 = 1$.
(ii) $\frac{1}{2}x + \frac{1}{12}\sinh 6x + c$, (ii) $\cosh^{-1}\frac{3}{2}$ or 0.9624.
4. (i) $y = \sqrt{\frac{1}{2(x^2 - c)}}$, (ii) $y = \frac{4e^x}{e^3(x+1)}$, (iii) $y = -x - \frac{1}{5} + c\,e^{5x}$.
5. (i) $y = Ae^{-7x} + Be^{2x}$, (ii) $y = e^{-x}(2\cos x - 2\sin x) - \sin 2x - 2\cos 2x$.
6. 0.4638
7. (i) $1 - \ln 2$
(ii) after converting to polars, integral becomes $\int_0^{2\pi}\int_1^2 r\sin r \, dr \, d\theta$.
8.

$$\begin{aligned} c_n &= \frac{1}{3}\int_0^3 f(t)e^{-2jn\pi t/3}\,dt \\ &= \frac{1}{3}\int_1^3 e^{-2jn\pi t/3}\,dt \quad \text{since } f(t) = 0 \text{ for } 0 < t < 1 \ \&\ f(t) = 1 \text{ for } 1 < t < 3 \\ &= \frac{1}{2jn\pi}\left(e^{-2jn\pi/3} - 1\right) \end{aligned}$$

Putting $n = 1$ and using $e^{j\theta} = \cos\theta + j\sin\theta$ gives result.