Spring 2010: Answers to EEE1018 Mathematics exam

1. (i) $(1-2x)^{-5} = 1 + 10x + 60x^2 + 280x^3 + \cdots$

(ii) $1 - 2x^2 + 2x^4 - \frac{4}{3}x^6 + \cdots$.
next bit obvious

$$\text{integral} \approx \int_0^{0.2} (-2 + 2x^2 - \frac{4}{3}x^4)\, dx = -0.395$$

2. (i) π, (ii) $\frac{1}{2}$, (iii) limit$= \lim_{y\to 0} \dfrac{\tan 2y}{y} = 2$

3. (i) $x = \tanh y = \dfrac{e^y - e^{-y}}{e^y + e^{-y}}$ gives first result. Then get

$$(1-x)e^{2y} = (1+x)$$

which gives next result. Finally set $x = (e^2-1)/(e^2+1)$.
(ii)

$$\text{integral} = \int \frac{dx}{\sqrt{(x-2)^2 + 16}}$$

which on substitution gives

$$\int \frac{dy}{\sqrt{y^2+1}} = \sinh^{-1} y + c = \sinh^{-1}\left(\frac{x-2}{4}\right) + c$$

4. (i) $y = A \exp\left(\frac{\sin 5x}{5}\right)$

(ii) $y = \tan(\frac{1}{2}x^2 + x + \frac{1}{4}\pi)$

(iii) integrating factor is x^2. Solution is $y = \dfrac{xe^x - e^x + c}{x^2}$.

5.
(i) $m = -5 \pm 2j$. General solution is $y = e^{-5x}(A\cos 2x + B\sin 2x)$. Solution satisfying the given initial conditions is

$$y = e^{-5x}\left(\cos 2x + \frac{7}{2}\sin 2x\right)$$

(ii) Complementary solution is $y_c = Ae^{3x} + Be^{-6x}$. General solution is

$$y = Ae^{3x} + Be^{-6x} + \frac{1}{5}e^{4x}$$

6. Let $f(x) = x^2 + \ln x - 2$. Then $f(1.2) = -0.38$ and $f(1.4) = 0.296$ so a root exists between 1.2 and 1.4.
Root is 1.3141.

7. (i)

$$\int_0^1 \int_0^{2x} (4 + y - 3x)\, dy\, dx = \int_0^1 \left[4y + \frac{y^2}{2} - 3xy \right]_{y=0}^{y=2x} dx = \int_0^1 (8x - 4x^2)\, dx = \frac{8}{3}$$

(ii)

$$\text{integral} = \int_{-\pi/2}^{\pi/2} \int_0^3 r^4 \cos\theta\, dr\, d\theta = \frac{486}{5}$$

8.

$$c_0 = \frac{1}{2\pi} \int_0^{2\pi} t\, dt = \pi$$

$$c_n = \frac{1}{2\pi} \int_0^{2\pi} t e^{-jnt}\, dt = -\frac{1}{jn}$$

$$\sum_{n=-1}^{1} c_n\, e^{jnt} = c_{-1}\, e^{-jt} + c_0 + c_1\, e^{jt} = \frac{1}{j}\, e^{-jt} + \pi - \frac{1}{j}\, e^{jt} = \pi - 2 \sin t$$

since $\sin t = (e^{jt} - e^{-jt})/2j$.