Mathematics 1b: Sheet 1 (will not be marked)

1. Find $\partial z/\partial x$ and $\partial z/\partial y$ when

(a) $z = x^2 \ln y$, (b) $z = x^2 e^{-3y}$, (c) $z = \dfrac{x}{x^2+y^2}$, (d) $z = \sin(x^2+5y)$, (e) $z = e^{xy}\ln y$

2. Find w_x, w_y and w_z when

$$\text{(a) } w = xyz - 2x^3 + y^2 - z, \quad \text{(b) } w = \frac{x^2y^3}{z}$$

3. Let $z = 2x^2y + xy^3 + 3x^4y^3$. Find all the first and second order partial derivatives $\dfrac{\partial z}{\partial x}$, $\dfrac{\partial z}{\partial y}$, $\dfrac{\partial^2 z}{\partial x^2}$, $\dfrac{\partial^2 z}{\partial y^2}$, $\dfrac{\partial^2 z}{\partial x \partial y}$ and $\dfrac{\partial^2 z}{\partial y \partial x}$ and check that $\dfrac{\partial^2 z}{\partial x \partial y} = \dfrac{\partial^2 z}{\partial y \partial x}$.

4. If k is a constant and $T = e^{-4kt}(\cos 2x + \cos 2y)$, calculate $\dfrac{\partial T}{\partial t}$, $\dfrac{\partial^2 T}{\partial x^2}$ and $\dfrac{\partial^2 T}{\partial y^2}$, and check that

$$\frac{\partial T}{\partial t} = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)$$

5. It is a general result that, for any function $z = f(x, y)$,

$$\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2$$

where r and θ are the polar coordinates of (x, y), so that $x = r\cos\theta$ and $y = r\sin\theta$. Show that the function in Question 1(c) becomes $z = \dfrac{\cos\theta}{r}$ in polar coordinates. For this particular function, check the correctness of the above equation.

6. In a coal processing plant a formula for the flow rate Q of slurry along a pipe is

$$Q = \frac{\pi p r^4}{8\eta l}$$

Calculate the partial derivatives of Q with respect to each of the four variables p, r, η and l and show that, in the usual notation,

$$\frac{\delta Q}{Q} \approx \frac{\delta p}{p} + 4\frac{\delta r}{r} - \frac{\delta \eta}{\eta} - \frac{\delta l}{l}.$$

If the four variables p, r, η and l are subject to maximum percentage errors of 2%, 1%, 3% and 2% respectively (so that, for example, $\left|\dfrac{\delta p}{p}\right| \leq 0.02$), calculate the maximum percentage error in Q.