UNIVERSITY OF SURREY©

Faculty of Engineering and Physical Sciences

Undergraduate Programmes in Engineering

Level 1

ENG1002 Mathematics 1b

Time allowed: 2 hours | Spring Semester 2011

Answer all questions. All working must be shown. Approved calculators may be used. *The marks for each question are shown in brackets; you should note that some questions carry more marks than others.*

1. Solve the following differential equations, expressing the solution in the form $y = f(x)$:

(i) $\dfrac{dy}{dx} = \dfrac{2x}{\sin y}$ [4]

(ii) $(x+1)\dfrac{dy}{dx} = y^2 + 1$ subject to $y(0) = 1$ [5]

(iii) $x\dfrac{dy}{dx} - 2y = \frac{1}{x}$ [5]

2. When an electric motor is under load, it generates heat internally at a constant rate H and also loses heat to its surroundings. The temperature difference T at time t between the motor and its surroundings satisfies

$$\frac{dT}{dt} = H - kT$$

where k is a constant. Find T as a function of t given that $T = 0$ when $t = 0$. What does T tend to as $t \to \infty$? [8]

3. Solve the differential equations:

(i) $\dfrac{d^2y}{dx^2} - 6\dfrac{dy}{dx} + 34y = 0$ [4]

(ii) $\dfrac{d^2y}{dx^2} - \dfrac{dy}{dx} - 2y = 7x - 2$ subject to $y(0) = 1$ and $y'(0) = -4$ [9]

4. Calculate the partial derivatives $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ of each of the following

(i) $z = 6x^3 - 2xy^2 + 4x^3y^4 + 2y + 3x$ [4]

(ii) $z = x\,e^{x/y}$ [4]

5. Let $z = 2x^2 + 2y^2 - x^2y + 2xy - 4x - 5y + 3$.

(i) Find the first and second order partial derivatives $\dfrac{\partial z}{\partial x}, \dfrac{\partial z}{\partial y}, \dfrac{\partial^2 z}{\partial x^2}, \dfrac{\partial^2 z}{\partial y^2}$ and $\dfrac{\partial^2 z}{\partial x \partial y}$. [5]

(ii) Show that z has stationary points at $(x, y) = (1, 1)$, $(3, 2)$ and $(-1, 2)$ and determine their nature. [9]

6. For each of the following systems of simultaneous equations determine whether the system has a unique solution, an infinite number of solutions or no solution and solve where possible. [14]

(i)
$$\begin{aligned} x - 2y - 4z &= -8 \\ 2x + 3y + 4z &= 10 \\ 5x + 4y + 4z &= 11 \end{aligned}$$

(ii)
$$\begin{aligned} x + 8y - 3z &= -6 \\ 2x + 4y + z &= -6 \\ 4x + 8y + 5z &= 6 \end{aligned}$$

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7.

(i) Find the rank of the matrix

$$\begin{pmatrix} 2 & 1 & 0 & 4 & 3 \\ -1 & 2 & 4 & -2 & 6 \\ 7 & -4 & -12 & 14 & -12 \end{pmatrix}$$ [5]

(ii) Show that

$$\begin{vmatrix} 1+a & a & a \\ a^2 & 1+a^2 & a^2 \\ a^3 & a^3 & 1+a^3 \end{vmatrix} = \frac{1-a^4}{1-a}$$ [5]

8. Find the moment of inertia, about the x-axis, of the solid object formed by rotating the lamina $0 \le y \le \sqrt{x}$, $0 \le x \le 4$, about the x-axis. You may leave your answer in terms of the density ρ (the mass per unit volume) of the object.
[You may assume that the moment of inertia of a disk of mass M, radius a, about an axis through its centre and perpendicular to the plane of the disk, is $\frac{1}{2}Ma^2$.] [7]

9. Evaluate the following double integrals:

(i) $\int_0^1 \int_0^1 (4 + x - y^2)\, dy\, dx$ [5]

(ii) $\iint_D e^{-x^2/3}\, dA$ where D is the triangle formed by the x-axis, the line $y = 2x$ and the line $x = 2$. [Hint: do the integration in the y direction first]. [7]

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