UNIVERSITY OF SURREY©

Faculty of Engineering and Physical Sciences

Undergraduate Programmes in Engineering

Level 1

ENG1002 Mathematics 1b

Time allowed: 2 hours   Spring Semester 2010

Answer all questions. All working must be shown. Approved calculators may be used. *The marks for each question are shown in brackets; you should note that some questions carry more marks than others.*

1. Solve the following differential equations for y, giving the answer in the form $y = f(x)$:

(i) $\frac{dy}{dx} = -2yx^2$, [3]

(ii) $\frac{dy}{dx} = (1 + y^2)e^x$, [4]

(iii) $\frac{dy}{dx} = e^{-x^2} - 2xy$ subject to $y(2) = 0$ [6]

2. The temperature in your house is 20^oC when, at midnight, the heating system breaks down. The temperature outside is 2^oC. After 30 minutes, the temperature inside the house has dropped to 14^oC. What will the temperature be at 1am? [Assume the temperature T inside the house is given by $dT/dt = -k(T-2)$ for a suitable value of k]. [7]

3. Solve the following differential equations for y:

(i) $\frac{d^2y}{dx^2} - 4y = 0$ subject to $y(0) = 2$ and $y'(0) = 5$ [7]

(ii) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 13y = \sin 2x$ [7]

4. Find the partial derivatives $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ of each of the following:

(i) $z = x + 3x^2y - 4xy^3 - \ln(y + 1)$ [4]

(ii) $z = \frac{x}{xy - 1}$ [4]

5. A surveyor is trying to calculate the height of a wall, which you can assume to be vertical. At a horizontal distance x from the wall, and from a height h above the horizontal ground, he measures the angle of elevation θ to the top of the wall. Show that the wall's height H is given by

$$H = h + x \tan\theta$$

The surveyor now tells you that he is certain as to the value of h but that his measurements of x and θ may be out by small amounts δx and $\delta\theta$ respectively. Show that his calculation of the height H may be out by an amount δH where

$$\delta H \approx \tan\theta\,\delta x + x\,\sec^2\theta\,\delta\theta$$

The surveyor measures x to be 30 ± 0.1 metres and θ to be 20 ± 0.2 degrees. If $h = 1$ metre calculate H and δH. [NB: make sure you do the calculations in radians]. [7]

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6. A rectangular building of length x, breadth y and height z is to have a volume of 8000 cubic metres. It is desired to minimise its annual heating costs, which amount to £1 per square metre for the roof and each of the two walls of area yz, and £2 per square metre for the remaining two walls. Show that the annual heating cost of the building is given by

$$C(x, y) = xy + \frac{16000}{x} + \frac{32000}{y}$$

What dimensions of the building will minimise the annual heating costs? *You must confirm that you have a minimum.* [12]

7. Write the system of simultaneous equations

$$\begin{aligned} x + y + 2z &= -7 \\ -4x - y + 3z &= 1 \\ 2x + y - z &= -1 \end{aligned}$$

in matrix form [2 marks]. Solve the system and check that the values you have found fit all three equations. [2 marks for each correct value, provided you have shown the working]. [8]

8. Let A be the matrix

$$A = \begin{pmatrix} 3 & 2 & -1 \\ 2 & -4 & 3 \\ 5 & 2 & 1 \end{pmatrix}$$

Find A^2 and A^3 and show that $A^3 - 18A + 28I$ is the zero matrix. [7]

9. (i) Find the rank of the matrix

$$\begin{pmatrix} -3 & 2 & 5 & 3 \\ 4 & -2 & 1 & -2 \\ 1 & 0 & 6 & 1 \\ -2 & 2 & 11 & 4 \end{pmatrix}$$ [6]

(ii) Show that

$$\begin{vmatrix} 1+b & a & a^2 \\ 1 & a+b & a^2 \\ 1 & a & a^2+b \end{vmatrix} = b^2(1 + a + a^2 + b)$$ [6]

10. Evaluate the following double integrals:

(i) $\displaystyle\int_0^2 \int_0^{\pi/4} y^3 \cos 2x \, dx \, dy$ [5]

(ii) $\displaystyle\iint_D \frac{x^2}{y} \, dA$, where D is the region enclosed between the curve $y = e^x$ and the lines $x = 1$ and $y = 1$. [Hint: integrate in the y direction first]. Your solution should include a clear sketch of the region D. [7]

Internal Examiner: S.A. GOURLEY