UNIVERSITY OF SURREY©

School of Engineering

Undergraduate Programmes in Engineering

Level 1

SE 0102 Mathematics 1b

Time allowed: 2 hours Spring Semester 2006

Answer all questions. All working must be shown.
The marks for each question are shown in brackets; you should note that some questions carry more marks than others.

1. Solve the following differential equations for y, giving the answer in the form $y = f(x)$:

(i) $\dfrac{dy}{dx} = -xy$ [3]

(ii) $\dfrac{dy}{dx} = e^{2x-y}, \quad y(0) = 0$ [5]

(iii) $x\dfrac{dy}{dx} - y = -3x^2$ [5]

2. A cold beer initially at 35^0F warms up to 40^0F in 3 minutes while sitting in a room of temperature 70^0F. How warm will the beer be if left out for 20 minutes?
[Hint: the temperature $T(t)$ of the beer will satisfy a differential equation of the form $\dfrac{dT}{dt} = k(70 - T)$.] [7]

3. Solve the following differential equations for y:

(i) $\dfrac{d^2y}{dx^2} - 4\dfrac{dy}{dx} + 13y = 0$ [4]

(ii) $\dfrac{d^2y}{dx^2} - \dfrac{dy}{dx} - 2y = e^{-3x}, \qquad y(0) = 1, \qquad y'(0) = -1$ [9]

4. Find the partial derivatives $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ of each of the following:

(i) $z = x^2y + 4xy^3$ [4]

(ii) $z = x\ln(x + \sin y)$ [4]

5. Let $z = 3xy - x^3 - y^3 + 2$.

(i) Find the first and second order partial derivatives $\dfrac{\partial z}{\partial x}, \dfrac{\partial z}{\partial y}, \dfrac{\partial^2 z}{\partial x^2}, \dfrac{\partial^2 z}{\partial y^2}$ and $\dfrac{\partial^2 z}{\partial x \partial y}$. [5]

(ii) Show that z has stationary points at $(x, y) = (0, 0)$ and $(1, 1)$ and determine their nature. [8]

6. Write the system of simultaneous equations

$$\begin{aligned} 2x - 3y + z &= 4 \\ x + 2y - 2z &= 1 \\ 3x + y - 2z &= -2 \end{aligned}$$

in matrix form [2 marks], and hence solve the system [2 marks for each correct value, provided you have shown the working]. [8]

[SEE NEXT PAGE

7. Let A be the matrix

$$A = \begin{pmatrix} 3 & 2 & 2 \\ 2 & 2 & 0 \\ 2 & 0 & 4 \end{pmatrix}$$

Find A^2 and A^3 and show that $A^3 - 9A^2 + 18A$ is the zero matrix. [7]

8. (i) Find the rank of the matrix

$$\begin{pmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \\ -1 & 3 & 2 & 2 \end{pmatrix}$$ [6]

(ii) Show that

$$\begin{vmatrix} b+c & c+a & b+a \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix} = 0$$ [6]

9. Evaluate the following double integrals:

(i) $\int_0^{\frac{\pi}{4}} \int_0^1 y \cos 2x \, dy \, dx$ [5]

(ii) $\iint_D (x^2 + y) \, dA$ where D is the region between the curves $y = x^3$ and $y = x^2$ for $0 \leq x \leq 1$. [7]

10. A thin plate of constant density occupies the region bounded by the x-axis, the y-axis and the line $y = 4 - 2x$. Sketch this region and find the coordinates of its centre of mass. [7]

Internal Examiner: S.A. GOURLEY