UNIVERSITY OF SURREY©

School of Engineering

Undergraduate Programmes in Engineering

Level 1

SE 0102 Mathematics 2

Time allowed: 2 hours Spring Semester 2003

Answer all questions. All working must be shown.
The marks for each question are shown in brackets; you should note that some questions carry more marks than others.

1. Solve the following differential equations, expressing the solution in the form $y = f(x)$:

(i) $\dfrac{dy}{dx} = \dfrac{x^2+2}{y}$ [4]

(ii) $\dfrac{dy}{dx} = \dfrac{y+1}{x+3}$ subject to $y(0) = 2$ [5]

(iii) $\dfrac{dy}{dx} + \dfrac{3}{x}y = 1$ [5]

2. Dr Gourley likes to consume a beer after his Friday afternoon lecture to the engineering students. At 3pm he removes the beer from the fridge, which is at temperature 3^oC, and places it in his office which is at temperature 22^oC. At 3.07pm the temperature of the beer has risen to 6^oC. Dr Gourley enjoys his beer best when it has temperature 11^oC. When (to the nearest minute) would you advise him to start drinking it?
[Assume that the temperature T of the beer satisfies $dT/dt = k(22 - T)$ for some number k]. [8]

3. Solve the differential equations:

(i) $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + 17y = 0$ [4]

(ii) $\dfrac{d^2y}{dx^2} - 4\dfrac{dy}{dx} + 3y = 5e^{-2x}$ subject to $y(0) = 1$ and $y'(0) = -1$ [9]

4. Calculate the partial derivatives $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ of each of the following

(i) $z = 4x^2 - y + 3x^3y^4$ [4]

(ii) $z = \dfrac{x}{x^2+y^2}$ [4]

5. Let $z = x^2 + 2y^2 - x^2y$.

(i) Find the first and second order partial derivatives $\dfrac{\partial z}{\partial x}$, $\dfrac{\partial z}{\partial y}$, $\dfrac{\partial^2 z}{\partial x^2}$, $\dfrac{\partial^2 z}{\partial y^2}$ and $\dfrac{\partial^2 z}{\partial x \partial y}$. [5]

(ii) Show that z has stationary points at $(x, y) = (0, 0)$, $(2, 1)$ and $(-2, 1)$ and determine their nature. [9]

6. For each of the following systems of simultaneous equations determine whether the system has a unique solution, an infinite number of solutions or no solution and solve where possible. [14]

(i)
$$\begin{aligned} x - 2y - 2z &= 1 \\ 2x + 3y + 2z &= 2 \\ 5x + 4y + 2z &= 4 \end{aligned}$$

(ii)
$$\begin{aligned} 2x - y + z &= -1 \\ x + 4y - 3z &= -8 \\ 4x + 4y + 5z &= 34 \end{aligned}$$

[**SEE NEXT PAGE**

7.

(i) Find the rank of the matrix

$$\begin{pmatrix} -2 & 5 & 7 \\ 0 & 1 & -3 \\ -4 & 11 & 11 \end{pmatrix} \qquad [5]$$

(ii) Show that

$$\begin{vmatrix} 1+a & a & a \\ b & 1+b & b \\ c & c & 1+c \end{vmatrix} = 1+a+b+c \qquad [5]$$

8. A thin plate of constant density occupies the region bounded by the x-axis and the curve $y = \cos x$, $-\frac{1}{2}\pi \leq x \leq \frac{1}{2}\pi$. Sketch this region and find the coordinates of the centre of mass of the plate. [7]

9. Evaluate the following double integrals:

(i) $\int_0^1 \int_0^1 (2 - x - y)\, dy\, dx$ [5]

(ii) $\iint_D e^{-y^2/2}\, dA$ where D is the triangle formed by the y-axis, the line $2y = x$ and the line $y = 1$. [Hint: do the integration in the x direction first]. [7]

Internal Examiner: S.A. GOURLEY