Numerical Solutions for PDEs: Lab 3

1. Implement the θ-method for the heat diffusion equation on $[0, 1]$ with zero boundary conditions and initial condition $u(x, 0) = \sin \pi x$. Use the function "spdiags" to build the matrices. Choose values of θ, Δx and Δt in order to produce

- a stable accurate solution,
- a stable inaccurate solution,
- an unstable solution.

2. Adapt the FTCS code of Lab2 to produce a BTCS approximation, then a CTCS scheme (take zero boundary conditions and initial condition $u(x, 0) = \sin \pi x$.

3. Implement the Dufort Frankel method using:

- "for" loops together with the formula

$$U_{j,n+1} = \frac{1-2r}{1+2r} U_{j,n-1} + \frac{2r}{1+2r}(U_{j+1,n} + U_{j-1,n}).$$

- show that the scheme can be written in matrix form as

$$\mathbf{U}^{n+1} = \mathbf{A}_1 \mathbf{U}^{n-1} + \mathbf{A}_2 \mathbf{U}^n + \mathbf{b},$$

  where $\mathbf{U}^n$ is the vector

$$\mathbf{U}^n = \begin{pmatrix} U_{1,n} \\ \vdots \\ U_{N-1,n} \end{pmatrix},$$

  $\mathbf{A}_1$ is the matrix

$$\mathbf{A}_1 = \frac{1-2r}{1+2r} \text{eye}(N-1, N-1)$$

  $\mathbf{A}_2$ is the matrix

$$\mathbf{A}_2 = \frac{2r}{1+2r} \text{diag}([1, 1], [-1, 1])$$

  and $\mathbf{b}$ is the vector

$$\mathbf{U}^n = \begin{pmatrix} U_{0,n} \\ 0 \\ \vdots \\ 0 \\ U_{N,n} \end{pmatrix},$$