Numerical Solutions for PDEs: Lab 2

We consider the heat diffusion in one spatial dimension, ie

$$u_t = u_{xx}, \quad x \in (0,1), \quad t > 0, \tag{1}$$

with boundary conditions

$$u(0,t) = a(t)\ ,\ u(1,t) = b(t)$$

and initial condition $u(x,0) = f(x)$. Recall that the FTCS approximation for this problem is given by

$$U_{i,j+1} = rU_{i+1,j} + (1-2r)U_{i,j} + rU_{i-1,j}, \tag{2}$$

and we described the following algorithm to solve numerically the problem:

Algorithm :

1. Choose N and r and find Δx and Δt.
2. Use the initial conditions to find $U_{i,0}$, $i = 0, \dots, N$.
3. Use equation (2) to calculate $U_{i,j+1}$ from $U_{:,j}$ for all $i = 1, \dots, N-1$, and use the boundary conditions to calculate $U_{0,j+1}$ and $U_{1,j+1}$.
4. Set $j = j+1$ and repeat 3. and 4. until the desired time is reached.

1. Implement this algorithm with matlab, choose zero boundary conditions and $u(x,0) = \sin(\pi x)$ as initial condition. Your solution should be stored in a 2 dimensional array U of size $(N+1) \times t_{\text{final}}$. Plot your solution using the function "surf", then plot it using the function "contour", finally using the "subplot" command plot the two graphs on a same page.

2. Adapt your matlab script to consider the following initial condition

$$u(0,x) = (1/2)(x-1)^2,$$

and a flux boundary condition at $x = 0$ given by $u_x(0,t) = 1$.

3. Use the functions "getframe" and "movie" to produce an animation of your solution.