CHARACTERIZATION OF ACOUSTIC CHANNEL IN NOISY SHALLOW OCEAN ENVIRONMENT USING A RAO-BLACKWELLIZED PARTICLE FILTER

X. Zhong[†], *V. N. Hari*[∘], *W. Wang*[⋆], *A. B. Premkumar*[#] *and C. T. Lau*[†]

[†][∘] CEMNET, School of Computer Engineering, College of Engineering, Nanyang Technological University, 639798, Singapore.

[⋆] Centre for Vision, Speech and Signal Processing, Department of Electronic Engineering, University of Surrey, UK, GU2 7XH.

[#]Electrical Engineering Department, Faculty of Engineering, University of Malaya, Kuala Lumpur, Malaysia.

[†][#]{xhzhong, asannamalai and asctlau}@ntu.edu.sg, [∘]harivishnu@gmail.com, and [⋆]w.wang@surrey.ac.uk

ABSTRACT

Acoustic signals in a shallow ocean environment are severely distorted due to the time-varying and inhomogeneous nature of the propagation channel. In this paper, a state-space model is introduced to characterize the uncertainties of the shallow ocean and a Rao-Blackwellized particle filter (RBPF) is developed to estimate the model parameters. Since both modal functions and horizontal wave numbers of the channel are assumed unknown, the state-space model has a high nonlinearity and high dimensionality. As the modal functions are linear with the measurements conditioning on the horizontal wave numbers, a Kalman filtering (KF) is employed to marginalize out the modal functions. Hence only the horizontal wave numbers need to be estimated by using a PF. Simulation results show that the proposed RBPF algorithm significantly outperforms the existing approaches.

Index Terms— Shallow ocean acoustic model, wave number, modal function, Rao-Blackwellized particle filter.

I. INTRODUCTION

Modeling acoustic wave propagation in a shallow ocean environment is an important topic and lies at the heart of many underwater signal processing applications. It is a challenging problem as uncertainties arise due to the time-varying and inhomogeneous nature of the ocean environment, and the received signal is seriously distorted due to multiple reflections from ocean boundaries.

Traditionally, matched-field processor (MFP) that compares the measured pressure-field to that predicted by a propagation model has been employed for parameter estimation [1]. To implement an MFP, the source location is required and is obtained by computing model predictions of the field at the array for various assumed source positions. However, the shallow ocean acoustic channel tends to vary with space and time. In [2], a state-space model was formulated to characterize all these dynamics and accordingly, extended Kalman filter (EKF) and unscented Kalman filter (UKF) approaches were introduced to estimate the states that characterize the channel. Such approaches are categorized as model-based methods in that the acoustic propagation and measurement uncertainties are modeled and included into the estimation algorithms. Very recently, the particle filtering (PF) approach which is more appropriate for nonlinear and non-Gaussian systems has been employed for parameter estimation of the shallow ocean model [2]–[4] and shown to achieve better estimation performance.

In a shallow ocean model, the horizontal wave numbers are considered unknown parameters in addition to the modal functions due to inhomogeneous time-varying nature of the environment. However, if all these parameters are blindly encapsulated into the estimated state, the performance of PF algorithm suffers due to the high dimensionality of the state. In this paper, a Rao-Blackwellized particle filtering (RBPF) method is developed to reduce the dimensionality of the state to be processed by PF. In essence, in this method the system model is reorganized in a linear fashion conditioned on the horizontal wave numbers. The Kalman filter (KF) is then employed to provide an optimal solution for this linear part. Following this, the horizontal wave numbers are the only state to be estimated by the PF. Hence, the PF can achieve better estimation accuracy with the same number of particles because the dimensionality is reduced. Our contribution here is incorporating a Rao-Blackwellization technique to obtain an analytical solution for part of the state, and therefore reduce the dimensionality of the state for PF algorithm. Simulations are organized to demonstrate the superiority of the proposed algorithm over the existing EKF and PF.

II. SIGNAL MODEL

For modeling acoustic wave propagation in a shallow ocean environment, we assume a horizontally stratified ocean of depth h with a known horizontal source range r_s and depth z_s, and the acoustic wave propagation is governed by the Helmholtz equation [5]. The normal-mode acoustic pressure propagation model can be written as

$$s(r_s, z) = \sum_{m=1}^{M} \beta_m(r_s, z_s)\psi_m(z) \tag{1}$$

where $\beta_m(r_s, z_s)$ is the modal coefficient defined as

$$\beta_m(r_s, z_s) = q\psi_m(z_s)\frac{e^{-\alpha_r(m)r_s}}{\sqrt{\kappa_r(m)r_s}}e^{j\kappa_r(m)r_s} \qquad (2)$$

where q is the source amplitude, and $\alpha_r(m)$, $\kappa_r(m)$ and $\psi_m(\cdot)$ are the modal attenuation, the horizontal wave number and the modal function associated with the mth mode respectively. The modal function holds an eigenvalue equation in z, given as [6]

$$\frac{\mathrm{d}^2}{\mathrm{d}z^2}\psi_m(z) + \kappa_z^2(m)\psi_m(z) = 0 \qquad (3)$$

for $m = 1, \ldots, M$. The eigen-value set $\{\psi_m(z)\}$ are the modal functions and κ_z is the wave number in the depth direction. The solutions of (3) depend on the sound speed profile $c(z)$, the boundary conditions and the corresponding dispersion relation given by

$$\kappa^2 = \frac{\omega^2}{c^2(z)} = \kappa_r^2(m) + \kappa_z^2(m) \qquad (4)$$

where $\kappa_r(m)$ is the mth horizontal wave number and ω is the harmonic source frequency. Let $\phi_m(z) = [\phi_{m1}(z), \phi_{m2}(z)]^T$ and define $\phi_{m1}(z) = \psi_m(z)$ and $\phi_{m2}(z) = \mathrm{d}/\mathrm{d}z\big(\psi_m(z)\big)$. The eigenvalue equation in state-space form is

$$\frac{\mathrm{d}}{\mathrm{d}z}\phi_m(z) = \mathbf{A}_m(z)\phi_m(z) \qquad (5)$$

where $\mathbf{A}_m(z)$ is the coefficient matrix given as

$$\mathbf{A}_m(z) = \begin{bmatrix} 0 & 1 \\ -\kappa_z^2(m) & 0 \end{bmatrix} \qquad (6)$$

The horizontal wave number $\kappa_r(m)$ can be roughly estimated using wavenumber spectrum estimation methods. However, these are unable to account for the fluctuating and time-varying nature of the environment. Thus $\kappa_r(m)$ is considered as an unknown environmental parameter to be estimated and is included into the state vector. Let $\theta_m(z) = \kappa_r(m)$. The whole state vector can be constructed as $\mathbf{x}(z) = [\mathbf{x}_\phi(z)^T, \mathbf{x}_\theta(z)^T]^T$, where

$$\mathbf{x}_\phi(z) = [\phi_1^T(z), \phi_2^T(z), \ldots, \phi_M^T(z)]^T \in \mathbb{R}^{2M\times 1} \qquad (7)$$

$$\mathbf{x}_\theta(z) = [\theta_1(z), \theta_2(z), \ldots, \theta_M(z)]^T \in \mathbb{R}^{M\times 1} \qquad (8)$$

The state process can be written as

$$\frac{\mathrm{d}}{\mathrm{d}z}\mathbf{x}_\phi(z) = \mathbf{A}(z)\mathbf{x}_\phi(z) + \mathbf{v}_\phi(z) \qquad (9)$$

$$\frac{\mathrm{d}}{\mathrm{d}z}\mathbf{x}_\theta(z) = \mathbf{0} + \mathbf{v}_\theta(z) \qquad (10)$$

where $\mathbf{A}(z) = \mathrm{diag}\big(\mathbf{A}_1(z), \ldots, \mathbf{A}_M(z)\big)$, and $\mathbf{v}_\phi(z)$ and $\mathbf{v}_\theta(z)$ are the zero-mean Gaussian processes given by $\mathbf{v}_\phi(z) \sim \mathcal{N}(0, \Sigma_\phi)$ and $\mathbf{v}_\theta(z) \sim \mathcal{N}(0, \Sigma_\theta)$ respectively. The measurement process can be written as

$$y(z) = s(r_s, z) = \mathbf{C}\big(\mathbf{x}_\theta(z)\big)\mathbf{x}_\phi(z) + \mathrm{w}(z) \qquad (11)$$

where $\mathbf{C}\big(\mathbf{x}_\theta(z)\big) = [\beta_1(\cdot), 0, \beta_2(\cdot), 0, \ldots, \beta_M(\cdot), 0]$, and $\mathrm{w}(z) \sim \mathcal{N}(0, \sigma^2)$ is the measurement noise process. Equations (9), (10) and (11) give a full state space model that describes the dynamics and uncertainties of the shallow ocean characteristics such as horizontal wave numbers and modal functions. In next section, an RBPF approach will be developed to estimate these characteristics.

III. RAO-BLACKWELLIZED PARTICLE FILTERING IMPLEMENTATION

Assume that z_ℓ is the depth of the ℓth sensor and $z_{1:\ell} = [z_1, \ldots, z_\ell]$. Given a measurement sequence $\mathbf{y}(z_{1:\ell})) = [y(z_1), \ldots, y(z_\ell)]$, the task is to estimate the posterior distribution $p(\mathbf{x}(z)|\mathbf{y}(z_{1:\ell}))$. Such a task can be achieved by using a Bayesian recursive estimation, given as

$$\begin{aligned} &\mathrm{Predict:} \\ &\quad p\big(\mathbf{x}(z_\ell)\big|\mathbf{y}(z_{1:\ell-1})\big) = \int \big(\mathbf{x}(z_\ell)\big|\mathbf{x}(z_{\ell-1}) \\ &\qquad\qquad p\big(\mathbf{x}(z_{\ell-1})\big|\mathbf{y}(z_{1:\ell-1})\big)\mathrm{d}\mathbf{x}(\mathrm{z}_{\ell-1}); \qquad (12) \\ &\mathrm{Update:} \\ &\quad p\big(\mathbf{x}(z_\ell)\big|\mathbf{y}(z_{1:\ell})\big) \propto p\big(\mathbf{y}(z_\ell)\big|\mathbf{x}(z_\ell)\big) \\ &\qquad\qquad p\big(\mathbf{x}(z_\ell)\big|\mathbf{y}(z_{1:\ell-1})\big). \qquad (13) \end{aligned}$$

In this recursion, $p\big(\mathbf{y}(z_\ell)\big|\mathbf{x}(z_\ell)\big)$ is the likelihood of the state and $p\big(\mathbf{x}(z_\ell)\big|\mathbf{y}(z_{1:\ell-1})\big)$ is the probability density function (PDF) of a prior distribution. The Bayesian recursion states that given the transition density and likelihood, the posterior distribution of the state can be recursively estimated.

The measurement function is nonlinear and therefore, the EKF approach [6] and PF approach [4] have been employed to estimate the posterior distribution. It has been shown that the PF approach is more appropriate under such a highly nonlinear scenario [7]. However, taking all states into account is cumbersome for the PF approach due to the curse of the dimensionality. Recall the state space model, the state of modal function $\mathbf{x}_\phi(z)$ holds a linear relationship with the measurement conditioning on the state $\mathbf{x}_\theta(z)$. This means given the estimation of $\mathbf{x}_\theta(z)$, an analytical solution for $p\big(\mathbf{x}_\phi(z)\big|\mathbf{x}_\theta(z), \mathbf{y}(z_{1:\ell})\big)$ can be obtained. Hence, it is possible to exploit a Kalman filter to marginalize out the modal functions. Consequently, only the horizontal wave numbers need to be handled by using the PF. Such a technique is referred to as Rao-Blackwellization and widely used for the state estimation where part of state space model is linear and Gaussian [7]. Using Bayesian theorem, the posterior distribution can be decomposed as

$$\begin{aligned} p\big(\mathbf{x}(z_\ell)\big|\mathbf{x}(z_{\ell-1}), \mathbf{y}(z_\ell)\big) &= \underbrace{p\big(\mathbf{x}_\theta(z_\ell)\big|\mathbf{y}(z_\ell)\big)}_{\text{PF}} \\ &\times \underbrace{p\big(\mathbf{x}_\phi(z_\ell)\big|\mathbf{x}_\theta(z_\ell), \mathbf{x}_\phi(z_{\ell-1}), \mathbf{y}(z_\ell)\big)}_{\text{KF}}, \qquad (14) \end{aligned}$$

in which $p\big(\mathbf{x}_\phi(z_\ell)\big|\mathbf{x}_\theta(z_\ell), \mathbf{x}_\phi(z_{\ell-1}), \mathbf{y}(z_\ell)\big)$ is analytically tractable and $p\big(\mathbf{x}_\theta(z_\ell)\big|\mathbf{y}(z_\ell)\big)$ can be estimated by PF approximation. Since part of the state can be estimated by using a KF, the dimension of the state to be processed by the PF can be reduced. Consequently, the Rao-Blackwellization based PF is able to provide better estimates than the standard PF when the same number of particles is used.

The core idea of PF is that it uses a set of particles and importance weights of these particles to approximate the posterior distribution. Assuming that N particles are used to approximate the above Bayesian recursion, the PDF $p\big(\mathbf{x}_\theta(z_\ell)\big|\mathbf{y}(z_\ell)\big)$ is represented by $\{\mathbf{x}_\theta^{(i)}(z_\ell), w_k^{(i)}\}_{i=1}^N$. The entire procedure of PF processing can be summarized as following. At each time step, the particles are sampled according to the state dynamic model (10), given as

$$\mathbf{x}_\theta^{(i)}(z_\ell) \sim p\big(\mathbf{x}_\theta^{(i)}(z_\ell)\big|\mathbf{x}_\theta^{(i)}(z_{\ell-1})\big). \tag{15}$$

These particles are then employed in the KF steps to marginalize out the modal functions. Assume that at the previous time step, the state and covariance estimates are $\hat{\mathbf{x}}_\phi^{(i)}(z_{\ell-1})$ and $\hat{\mathbf{P}}_\phi^{(i)}(z_{\ell-1})$ respectively. The predictions are:

$$\mathbf{x}_\phi^{(i)}(z_\ell|z_{\ell-1}) = \mathbf{A}^{(i)}(z_\ell)\hat{\mathbf{x}}_\phi^{(i)}(z_{\ell-1}) \tag{16}$$

$$\mathbf{P}_\phi^{(i)}(z_\ell|z_{\ell-1}) = \mathbf{A}^{(i)}(z_\ell)\hat{\mathbf{P}}_\phi^{(i)}(z_{\ell-1})\mathbf{A}^{(i)}(z_\ell)^T + \Sigma_\phi \tag{17}$$

The Kalman gain is then calculated as

$$\mathbf{S}^{(i)} = \mathbf{C}\big(\mathbf{x}_\theta^{(i)}(z_\ell)\big)\mathbf{P}_\phi^{(i)}(z_\ell|z_{\ell-1})\mathbf{C}^T\big(\mathbf{x}_\theta^{(i)}(z_\ell)\big) + \sigma^2 \tag{18}$$

$$\mathbf{K}^{(i)} = \mathbf{P}_\phi^{(i)}(z_\ell|z_{\ell-1})\mathbf{C}^T\big(\mathbf{x}_\theta^{(i)}(z_\ell)\big)\big(\mathbf{S}^{(i)}\big)^{-1} \tag{19}$$

The dependency on z_ℓ in (18) and (19) is ignored to simplify the expression. The state and covariance are updated as

$$\hat{\mathbf{x}}_\phi^{(i)}(z_\ell) = \mathbf{x}_\phi^{(i)}(z_\ell|z_{\ell-1}) + \mathbf{K}^{(i)} \times \Big(y(z_\ell) - \mathbf{C}\big(\mathbf{x}_\theta^{(i)}(z_\ell)\big)\mathbf{x}_\phi^{(i)}(z_\ell|z_{\ell-1})\Big) \tag{20}$$

$$\hat{\mathbf{P}}_\phi^{(i)}(z_\ell) = \mathbf{P}_\phi^{(i)}(z_\ell|z_{\ell-1}) - \mathbf{K}^{(i)} \times \mathbf{C}\big(\mathbf{x}_\theta^{(i)}(z_\ell)\big)\mathbf{P}_\phi^{(i)}(z_\ell|z_{\ell-1}) \tag{21}$$

The filtered distribution is $\mathcal{N}\big(\hat{\mathbf{x}}^{(i)}(z_\ell), \hat{\mathbf{P}}^{(i)}(z_\ell)\big)$. The importance weights of the particles are then evaluated by

$$w_k^{(i)} = w_{k-1}^{(i)} p\big(y(z_\ell)\big|\mathbf{x}_\theta^{(i)}(z_\ell)\big), \tag{22}$$

The likelihood of the particles are then calculated as

$$p\big(y(z_\ell)\big|\mathbf{x}_\theta^{(i)}(z_\ell)\big) = \frac{1}{\sqrt{2\pi\sigma^2}} \times \exp\left\{-\frac{1}{2\sigma^2}\Big(y(z_\ell) - \mathbf{C}\big(\mathbf{x}_\theta^{(i)}(z_\ell)\big)\mathbf{x}_\phi^{(i)}(z_\ell|z_{\ell-1})\Big)^2\right\} \tag{23}$$

Algorithm 1: RBPF for model parameter estimation.

Initialisation: for $i = 1, \ldots, N$, draw particles according to (26); set the initial weight $\tilde{w}_0^{(i)} = 1/N$;
for $\ell \leftarrow 1$ **to** L **do**
  for $i \leftarrow 1$ **to** N **do**
    1) draw samples according to equation (15);
    2) KF marginalization from (16) to (21);
    3) compute the likelihood according to (23);
    4) calculate the weight according to (22);
  end
  5) normalise the weight $\tilde{w}_k^{(i)} = w_k^{(i)} / \sum_{i=1}^N w_k^{(i)}$;
  6) resample the particles according to the weights;
  7) output the estimates.
end

After the resampling scheme, the posterior distribution of the state is thus approximated by

$$p\big(\mathbf{x}_\theta(z_\ell)\big|\mathbf{y}(z_\ell)\big) \approx \sum_{i=1}^N \tilde{w}_k^{(i)} \delta_{\mathbf{x}_\theta^{(i)}(z_\ell)}\big(\mathbf{x}_\theta(z_\ell)\big), \tag{24}$$

$$p\big(\mathbf{x}_\phi(z_\ell)\big|\mathbf{y}(z_\ell)\big) \approx \sum_{i=1}^N \tilde{w}_k^{(i)} \mathcal{N}(\hat{\mathbf{x}}^{(i)}(z_\ell), \hat{\mathbf{P}}^{(i)}(z_\ell)), \tag{25}$$

where $\delta(\cdot)$ is a Dirac-delta function, and $\tilde{w}_k^{(i)}$ is a normalized weight. In practical implementation, the state can be initialized by estimates via the MFP method. Assume that the initial state is $\bar{\mathbf{x}}_0$. The initial distribution can be given as

$$\mathbf{x}_0^{(i)} \sim \mathcal{N}(\bar{\mathbf{x}}_0, \mathbf{\Sigma}_0) \tag{26}$$

where $\mathbf{\Sigma}_0$ is the variance of initial distribution that characterizes the error of the MFP estimates. All implementation steps of the RBPF approach are summarized in Algorithm 1. The proposed approach differs from traditional PF [4] for model parameter estimation in that the modal functions are analytically estimated by using KF, and only the horizontal wave numbers are estimated by using PF.

IV. SIMULATIONS

In this section, simulations are provided to demonstrate the tracking performance. The performance is compared with that of existing EKF approach [6] and PF approach [4]. A noisy shallow ocean channel with a depth of 100 m is simulated. The signal source is located at a depth of 36 m and a horizontal range of 5 km. The center frequency of the source signal is 100 Hz. This leads to the signal field propagating with six normal modes. Considering the fluctuations in the ocean, the wave numbers are initialized with a bias of 1.0×10^{-4}. This accounts for the uncertainty present in describing the shallow ocean environment. Other parameters are set as following: $N = 200$, $\sigma_2 = 1.0 \times 10^3$, $\Sigma_\phi = 1.0 \times 10^{-6}$ and $\Sigma_\theta = 1.0 \times 10^{-8}$. These parameters

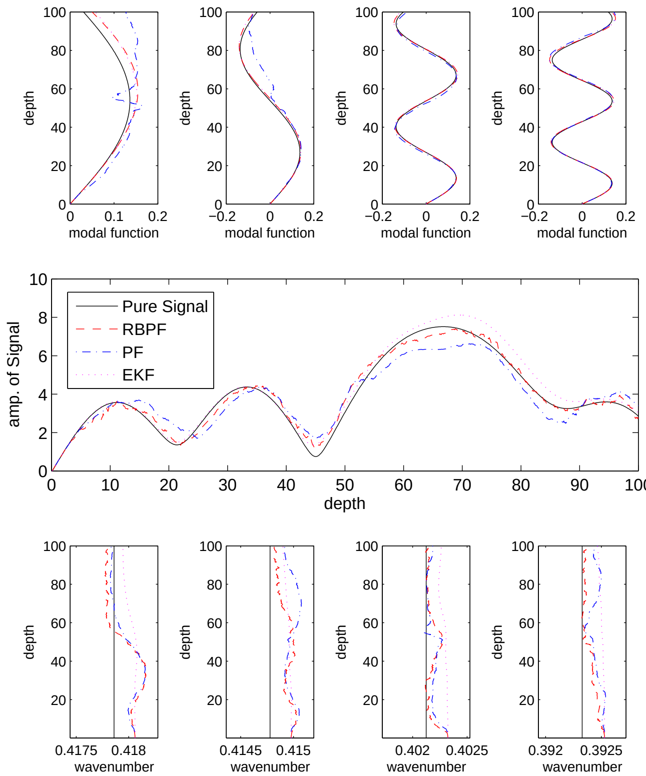

Fig. 1. Results of estimation and ground truth under SNR = 10 dB for: modes 1, 2, 4 and 5 (top), pressure field (middle) and corresponding wave numbers (bottom).

are selected based on extensive experimental study and are found be able to provide good estimates.

Figure 1 presents a comparison of estimation using the RBPF against the EKF and traditional PF at an SNR of 10 dB. The ground truth and corresponding estimates of modal functions 1, 2, 4 and 5 are plotted on the top of the figure. Signal field and wave numbers are also presented. The estimation results show that the proposed RBPF algorithm provides better accuracy than traditional EKF and PF approaches. RBPF converges faster to the true value of the wave numbers despite the initial bias. Its accuracy in estimating modal functions is significantly better. Consequently, the signal recovery is significantly improved in the noisy environment.

Multiple Monte Carlo (MC) simulations are also organized to study the performance of the proposed algorithm. The normalized mean square error (MSE) over 50 MC runs for the modal function, wave number and signal field estimation is presented in Fig. 2. Different noisy data from 0dB to 30dB with a 5dB increment are generated. The results

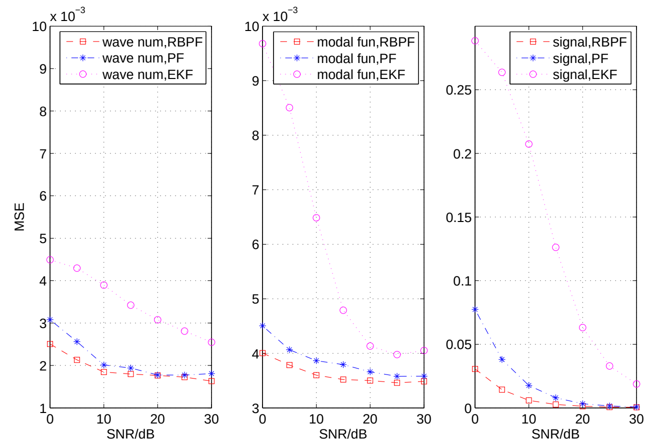

Fig. 2. RMSE versus different SNRs for wave numbers (left), modal functions (center) and signal field (right)

further demonstrate the superiority of the proposed algorithm over other existing approaches. For all estimations, the MSE is significantly lower than that of other existing approaches. Due to Rao-Blackwellization, the state dimensionality to be processed by the PF is reduced. Consequently, RBPF performs much better than the traditional PF approach.

V. CONCLUSION

An RBPF approach is introduced in this paper to estimate the modal functions and wavenumbers of a shallow ocean channel. Conditioning on the horizontal wave numbers, the modal functions are linearly dependent on the measurements. Hence, the modal functions are analytically estimated by using a KF and only the wave numbers need to be estimated by the PF. The estimation accuracy is thus improved by the proposed approach. Simulations show that the proposed RBPF algorithm significantly outperforms the existing approaches in estimating these parameters. Future work includes modal order detection and real underwater data applications.

VI. REFERENCES

[1] A. B. Baggeroer, W. A. Kuperman, and P. N. Mikhalevsky, "An overview of matched field methods in ocean acoustics," *IEEE J. Ocean. Eng.*, vol. 18, no. 4, pp. 401–424, 1993.

[2] C. Yardim, P. Gerstoft, and W. S. Hodgkiss, "Tracking of geoacoustic parameters using kalman and particle filters," *J. Acoust. Soc. Amer.*, vol. 125, no. 2, pp. 746–760, 2009.

[3] J. Candy, "Particle filtering for signal enhancement in a noisy shallow ocean environment," in *OCEANS 2010 IEEE*, Sep. 2010, pp. 1–6.

[4] ——, "An adaptive particle filtering approach to tracking modes in a varying shallow ocean environment," in

OCEANS, 2011 IEEE, Santander, Spain, Jun. 2011, pp. 1 –7.

[5] L. Bjorno, Ed., *Underwater Acoustics and Signal Processing*. Dordrecht: Reidel, 1981.

[6] J. Candy and E. Sullivan, “Model-based identification: an adaptive approach to ocean-acoustic processing,” *Ocean. Eng., IEEE J.*, vol. 21, no. 3, pp. 273 –289, jul 1996.

[7] B. Ristic, S. Arulampalam, and N. Gordon, *Beyond the Kalman Filter: Particle Filters for Tracking Applications*. Artech House, 2004.