Joint Array and Spatial Sparsity Based Optimisation for DoA Estimation

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***Abstract*—Traditional spatial sparse techniques for DoA estimation are implemented by full arrays. However in practice, it is desirable to use as few sensors as possible to reduce the cost for manufacturing the array or to counter against sensor failure. As a result, joint optimisation of sparse array and spatial sparsity becomes an ideal alternative. In most of existing methods, these two kinds of sparsity are studied separately. This paper proposes a joint model which achieves source detection in a subset of space using partial array sensors. The core idea is to use the weight coefficients obtained in sparse array optimisation to scale the model for the sparse reconstruction based DoA estimation. Compressive sensing based optimisation is used for both steps. The numerical results of DoA estimation for both stationary source and moving source are used to demonstrate the feasibility of this joint model.**

I. Introduction

An acoustic array system, which is a collection of acoustic transducers operating through array signal processing techniques, has been used in many areas, such as non-destructive evaluation, underwater detection and machine diagnosis. In these applications, Direction of Arrival (DoA) estimation from the array output is an important issue in source localisation and separation [1].

Classically, DoA estimation is addressed by methods, such as Capon beamformer, high-resolution and multiple signal classification (MUSIC) algorithm [2] [3] [4]. Recently, spatial sparsity motivated techniques, which aim at extracting meaningful lower-dimensional representations from high-dimensional data, have attracted wide interests in DoA estimation, since it drastically improves system performance on data storage and transmission [5]. Compressive sensing (CS) based sparse reconstruction [6], which is a popular solution to sparse representation, has been used for DoA estimation in [7], where the activity of source is assumed to be sparse and the sparsity is enforced by a constraint based on l_1 norm of a vector of the coefficients corresponding to the source activities in the spatial domain [7]. A sequential algorithm has been developed recently in [8] [9], where continuous DoA estimation is obtained from the array observation in an on-line manner, based on the least absolute shrinkage and selection operator (LASSO).

In the techniques discussed above, a full array is used and the adjacent sensor separation is often assumed to be no larger than half of the wavelength of the signal [10]. In practice, however it is desirable to use as few sensors as possible to reduce the cost for array design and manufacturing or to address the problem of sensor failure in the sense that the weight coefficients corresponding to the remaining working sensors can be re-configured with the sparse optimisation techniques. As a result, the idea of the sparse array was proposed as an alternative [11], where the CS techniques can also be used [12] by limiting the number of non-zero elements (l_0 norm) of the weighting parameters. Alternatively, the method can be more efficiently addressed as a minimisation of the l_1 norm of the weighting coefficients as in [13].

Spatial sparsity and array sparsity have been exploited separately in most existing methods discussed above. In [14], however, these two types of sparsity have been considered, where the direct data covariance (DDC) matrix is used to form a virtual array model with an extended aperture. Then two methods addressing the LASSO problem with this array model are presented. The DDC matrix instead of omni-directional steering vector is used to obtain the desired signal with the sparse array. However, the sparsity of the linear array was manually set in their experiments.

In this paper, we propose a new joint model to exploit sparsity in both array and source space where a CS based two-step iterative process is used. Through this joint model we are able to calculate the weight coefficients for each sensor, which are then used in a sequential sparse reconstruction algorithm to estimate the DoAs. Combining spatial sparsity and the sparse array optimisation enables DoA estimation in a lower-dimensional space and at the same time using as few sensors as possible.

This article is organised as follows. In Section II, the background about the sparse array and the spatial sparse representation is presented. The proposed algorithm is presented in Section III. In Section IV, numerical results on a stationary source, a moving source and noise added sources are presented. Finally, the conclusion is given in Section V.

II. Background

A. Array Model

We assume that a narrowband linear array is used, and each array element has an unitary sensitivity. In addition, source signals can only arrive in one half of the plane and the array is expected to have a perfect baffle, which means the arrival

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directions are from -90 degree to +90 degree along the plane of the array elements, and 0 degree is the normal to the line of the array.

As in [15], the observed sensor signal at each time step k is denoted as $\mathbf{y}_k = (y_{k1}, y_{k2}, \cdots, y_{kN})$, where N is the number of sensors. At each time step, there is a vector reflecting possible angles for the DoA $\mathbf{x}_k = (x_{k1}, x_{k2}, \cdots, x_{kM})$, where M is the number of potential source directions.

A matrix $\mathbf{A}$ is created by sampling all possible directions of arrivals (DoAs). The size of $\mathbf{A}$ is $N \times M$ where $N \ll M$. The nm-th element of $\mathbf{A}$ is defined by

$$\mathbf{A}_{nm} = \frac{1}{\sqrt{N}} exp\left[-j2\pi\mu_n \sin\theta_m\right] \tag{1}$$

where $\mu_n = \frac{d_n}{cT_s}$ with temporal sampling period T_s for $n = 1, 2, \cdots, N$, d_n denotes the distance between the n-th sensor and the middle sensor, c is the speed of wave propagation, and $\theta_m = \frac{\pi m}{M} - \pi/2$ is the DoA of the m-th hypothetical source to the n-th sensor in the array.

For each vector $\mathbf{x}_k$, there are M source directions corresponding to the columns of $\mathbf{A}$, leading to the array model

$$\mathbf{y}_k = \mathbf{A}\mathbf{x}_k + \mathbf{n}_k \tag{2}$$

where $\mathbf{n}_k$ is random noise. Here we consider isotropic noise in the assumed half plane as used in our experiments.

B. CS-Based Sparse Array

Following [13], the CS based sparse narrowband array problem can be formulated as a minimisation of the l_1 norm of the weight coefficients, as follows,

$$\begin{aligned} &\min \quad \|\mathbf{w}\|_1 \\ &subject \quad to \quad \|\mathbf{f} - \mathbf{w}^H\mathbf{A}\|_2 \le \alpha \end{aligned} \tag{3}$$

where $\mathbf{w} \in \mathbb{C}^N$ is the coefficient vector of the array, $\mathbf{f} \in \mathbb{C}^M$ is the vector holding the desired beam response at the sampled angular points θ_m for the frequency of interest Ω, $m = 1, 2, \cdots, M$, $\alpha \in \Re^+$ is the constraint to define how much the designed response is similar to the desired response, $(\cdot)^H$ is a Hermitian operator, $\|\cdot\|_1$ and $\|\cdot\|_2$ are respectively the l_1 and l_2 norm of their arguments.

In detail, $\mathbf{f}$ is defined by

$$\mathbf{f} = [f(\Omega, \theta_1), f(\Omega, \theta_2), \cdots, f(\Omega, \theta_M)] \tag{4}$$

where $f(\Omega, \theta)$ is a desired response at the direction θ.

C. Spatial Sparse Reconstruction Using LASSO

Assuming a full array is used, the DoA estimation problem can be cast as a sparse reconstruction problem. This can be achieved by dividing the source space into multiple ranges of possible directions, with each range assigned a weight to indicate the activity of the source in that rangle. To find the DoA of the source, the weights corresponding to each range need to be found. This can be achieved by e.g. using a sequential Bayesian technique based on the LASSO function, as in [8], [9], assuming that the number of active sources is small as compared to the total number of ranges of possible directions.

For the signal $\mathbf{y}_k$ at time k and source activation vector $\mathbf{x}_k$, the cost function to be minimised is given as follows,

$$\underset{\mathbf{x}_k}{\operatorname{argmin}} \quad \|\mathbf{y}_k - \mathbf{A}\mathbf{x}_k\|_2^2 + \mu\|\mathbf{D}\mathbf{x}_k\|_1 \tag{5}$$

with

$$\mathbf{D} = \sigma^2\mathbf{V} \tag{6}$$

$$\mathbf{V} = diag(\mathbf{v}) \tag{7}$$

where $\mathbf{D}$ and $\mathbf{V}$ are matrices holding the coefficients vector $\mathbf{v} = (v_1, v_2, \dots, v_{\mathrm{M}})^T$, which corresponds to the source activity in the source space, μ is a regularization parameter, and σ^2 is the noise variance.

As for sequential estimation, the result at the $(k+1)$-th time step should be calculated by the signal at the k-th time step. This is achieved based on the relationship between $v_m[k]$ and $v_m[k+1]$.

In the neighbourhood of an active source defined by a threshold l [9], the predicted weights $v_m[k+1]$ are calculated as below:

$$(v_m[k+1])^2 = \left(\sum_{j=-l}^{l} \frac{\alpha_j}{(v_{m+j}[k])^2}\right)^{-1} \tag{8}$$

with non-negative coefficients α_j, $\sum_j \alpha_j = 1$ and the weights in $\mathbf{v}$ are bounded by 1. In practice, a small threshold e.g. 100×2^{-52} is used to prevent the value of $v_{m+j}[k]$ from approaching zero in order to avoid invalid division.

When the estimation is out of the neighbourhood of the active source, the weight prediction is replaced by:

$$v_m[k+1] = v_m[k] + cv_0 \tag{9}$$

where v_0 and c are defined empirically [9].

III. Joint Sparsity Based Method

The proposed method consists of two main steps. With the input of a given beam, the first step is to implement the CS-based sparse array algorithm through the cost function (3) to obtain a series of sparse sensor weight coefficients. The second step is to plug the estimated weight coefficients into the cost function (5), as a scaling factor for $\mathbf{y}_k$.

For the CS-based sparse array optimisation with the cost function (3), as the vector weight coefficients $\mathbf{w}$ are complex numbers, i.e. $\mathbf{w} = \mathbf{w}_R + \mathbf{w}_I i$, to achieve sparse solutions, the l_1 norm should be modified as

$$\begin{aligned} &\min \quad \|\mathbf{w}_R\|_1 + \|\mathbf{w}_I\|_1 \\ &subject \quad to \quad \|\mathbf{f} - (\mathbf{w}_R + \mathbf{w}_I i)^H \mathbf{A}\|_2 \le \alpha \end{aligned} \tag{10}$$

where $\mathbf{f}$ is the input vector holding the desired beam response at source directions and α is a constraint controlling the fitting error [13].

After obtaining the estimated $\mathbf{w}$, the LASSO function (5) is then modified as

$$\underset{\mathbf{x}_k, v_m}{\operatorname{argmin}} \quad \|\mathbf{W}\mathbf{y}_k - \mathbf{A}\mathbf{x}_k\|_2^2 + \mu\|\mathbf{D}\mathbf{x}_k\|_1 \tag{11}$$

where $\mathbf{W}$ is defined as

$$\mathbf{W} = \begin{pmatrix} |w_1| & & \\ & \cdots & \\ & & |w_N| \end{pmatrix} \tag{12}$$

Both cost functions (10) and (11) are optimised by the CVX toolbox in Matlab [16].

The above two steps are iterated in an alternating manner. The input beam response $\mathbf{f}$ can be initialised randomly. The joint model is able to estimate the correct DoA estimation after a few iterations as follows,

1. Set k = 1 and create an initial beam response as input $\mathbf{f}$ of (10), corresponding to a randomly selected direction.
2. Implement two-step optimisation and obtain the possible angles $\mathbf{x}_k$ by solving (11).
3. Replace $\mathbf{f}$ by $(\mathbf{A}\mathbf{x}_k)^H\mathbf{A}$, and set $k = k$+1.
4. Repeat stages 2 to 3 until k reaches the predefined maximum time step K.

IV. Numerical Results

In this section, the performance of the proposed method is investigated for narrowband DoA estimation through a sparse linear array. All the experiments are based on simulated data, including a stationary source at 40 degree and a moving source starting from 0 degree, turning to 48, -48.5, 57, 52, 57, -57.5, 57 degrees, ending at 13.5 degree. Here we assume the source angle is changing at a constant range, hence no Doppler shift is considered in this paper.

The underwater speed of sound used in this model is assumed to be 1500 m/s, and the frequency of the sources is 200 Hz. In the noisy case, the level of noise in terms of Signal to Noise Ratio (SNR) is 20 dB. A grid of 100 potential sensors are used.

For the stationary source estimation, the inter-sensor spacing is 0.1λ (λ is the wavelength) and the maximum running step is $K = 20$. For the moving source estimation, the inter-sensor spacing is 0.05λ and the maximum running step is $K = 100$. Experiments show that the sensor separation should be small enough with respect to the scale of the array and running time. It was observed that the number of steps chosen for the stationary and moving source cases was large enough for the proposed algorithm to converge. The constraint value of α used in (10) is 0.3, which controls how close the designed response is to the desired response.

In order to measure the accuracy of DoA estimation, we define the performance metrics of Mean Square Error (MSE) and Standard Error (SE) according to functions (10) and (11) as follows,

$$MSE = 20log_{10}\left(\frac{\|\mathbf{f} - \mathbf{w}^H\mathbf{A}\|_2^2}{M}\right) dB \tag{13}$$

$$SE = \sqrt{\left(\frac{\|\mathbf{y}_k^H\mathbf{A} - (\mathbf{A}\mathbf{x}_k)^H\mathbf{A}\|_2^2}{M}\right)} degree \tag{14}$$

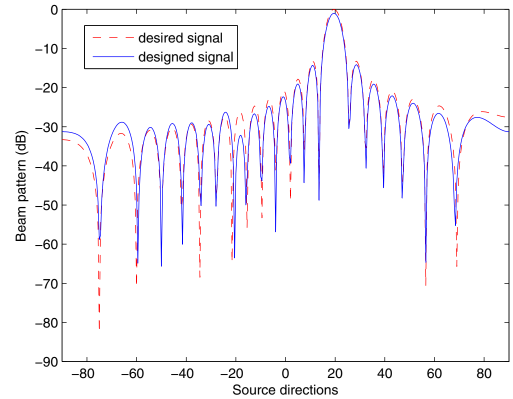

Fig. 1. The desired response $\mathbf{f}$ given in initialisation and the designed response $\mathbf{w}^H\mathbf{A}$ obtained by the first step of the proposed two-step method for the stationary source without noise.

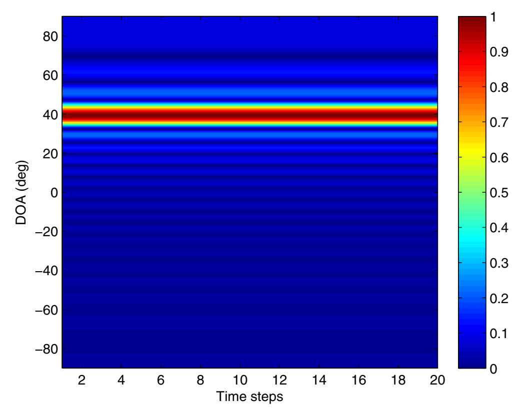

Fig. 2. DoAs estimated ($(\mathbf{A}\mathbf{x}_k)^H\mathbf{A}$) for the stationary source without noise, at 40 degree.

where M is the number of potential source directions. Both MSE and SE are calculated in each time step (the total time step is K) and the average values are calculated for each performance metric. We distinguish MSE in decibel and SE in degree because decibel is often used for presenting the difference in beam response while degree is used for measuring the DoA directions.

A. Stationary Source

The input beam response of the sparse array is initialised at 20 degree (intentionally set to be different from the ground truth of the source direction). It was observed in our experiments that 36 of 100 sensors become active in this case. The weight coefficients become stable after 20 time steps. Figure 1 illustrates the simulation between desired signal (the red dashed line) and designed signal (the blue line), where the desired response corresponds to $\mathbf{f}$ given in initialisation and designed response corresponds to $\mathbf{w}^H\mathbf{A}$ obtained by the first iteration of the proposed two-step method. The simulation shows the designed response matches well with the initial

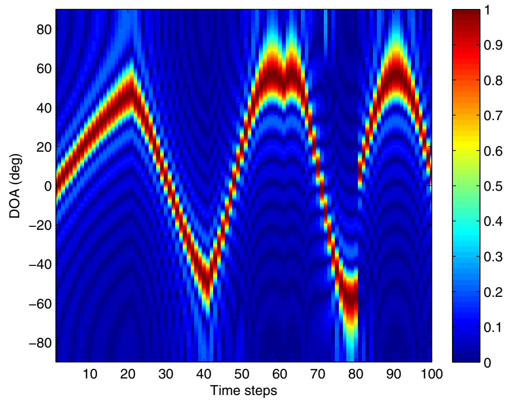

Fig. 3. DoAs estimated $((\mathbf{A}\mathbf{x}_k)^H\mathbf{A})$ for the moving source without noise.

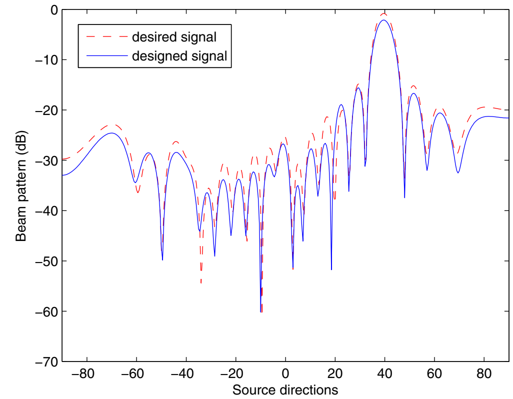

Fig. 4. The mean value of desired response $\mathbf{f}$ given in continuous k time steps and the mean value of designed response $\mathbf{w}^H\mathbf{A}$ obtained by the first step of the proposed two-step method for stationary source with noise (SNR = 20 dB).

desired response. The mean square error (MSE) defined by (13) between two responses is -66.7 dB.

With the result of the sparse array optimisation, the weight coefficients are then used to scale $\mathbf{y}_k$ in the LASSO problem (11), which is used to find spatial sparse representation $\mathbf{x}_k$.

In Figure 2, the source DoA at 40 degrees can be clearly observed through the beamforming of $((\mathbf{A}\mathbf{x}_k)^H\mathbf{A})$. The standard error (SE) defined by (14) between the estimated DoA $((\mathbf{A}\mathbf{x}_k)^H\mathbf{A})$ and the target DoA $(\mathbf{y}_k^H\mathbf{A})$ is 0.49 degrees, demonstrating a satisfactory performance of the joint method.

We should note that even though in the sparse array optimisation step, the sensors were selected based on the DoA initialised to an angle, e.g. 20 degree, which is different from the ground truth, the second step of the proposed algorithm is able to find the correct DoA based on the selected subset of the sensors. This is also the case observed in the moving source scenario, as discussed next.

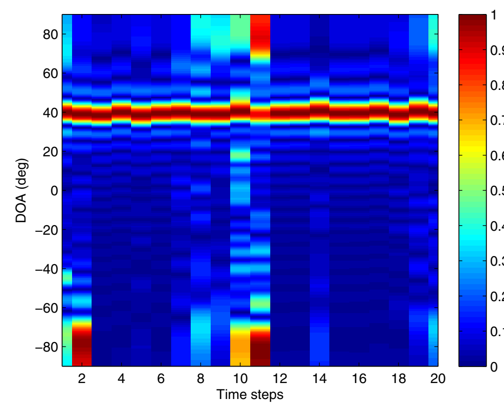

Fig. 5. DoAs estimated $((\mathbf{A}\mathbf{x}_k)^H\mathbf{A})$ for the stationary source with noise (SNR = 20 dB), at 40 degree.

N^{th}	Weights	N^{th}	Weights
6	3.1652	51	3.1769
7	0.9644	52	1.1229
16	0.2658	67	0.3500
17	0.4516	68	1.6212
18	0.8040	71	0.2090
29	0.2799	81	0.9776
33	2.2697	82	2.2576
34	0.3991	94	0.7304
40	0.3540	95	1.9352

TABLE I
WEIGHT COEFFICIENTS OF ARRAY FOR MOVING SOURCE WITH NOISE (SNR = 20 dB) AT TIME STEP OF 3.

B. Moving Source

For the moving source, the first input beam response of the sparse array is also initialised at 20 degree, then the source DoA estimated at time k is given as the input to the sparse array optimisation at time $k+1$. The optimised weight coefficients in sparse array changes in each time step. It was found that the average number of active sensors selected by the sparse array optimisation is 22 (note the total number of sensors is 100). The mean value of the MSE between the desired signal $\mathbf{f}$ and designed signal $\mathbf{w}^H\mathbf{A}$ is -70.6 dB for all the time steps.

Figure 3 shows the DoA estimates (in terms of $(\mathbf{A}\mathbf{x}_k)^H\mathbf{A})$ with respect to the time steps. The SE between the estimated DoA $((\mathbf{A}\mathbf{x}_k)^H\mathbf{A})$ and the target DoA $(\mathbf{y}_k^H\mathbf{A})$ is 0.27 degrees. Each turning point of the source can be seen clearly in this figure despite the existence of some estimation noise. We believe that the estimation noise can be further reduced with more time steps.

C. Signals with Random Noise

This section evaluates how the algorithm performs when the observed signals are noisy. To this end, noise at the level of SNR = 20 dB, which is isotropic around the half space and follows the i.i.d Gaussian distribution with zero mean and unit variance, is added to the observed signals. The initial input beam response is set the same as previous sections. It was

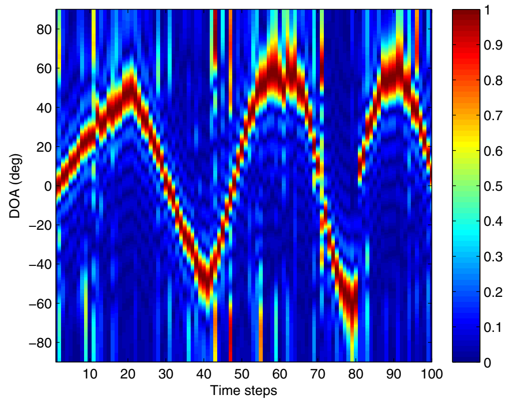

Fig. 6. DoAs estimated ($(\mathbf{A}\mathbf{x}_k)^H\mathbf{A}$) for the moving source with noise (SNR = 20 dB).

observed that, 22 active sensors were selected by the sparse array optimisation algorithm for the moving source, which was smaller than the 37 active sensors in the stationary case. The coefficients for optimised array change continuously due to the noise. Table I is an example of the weight coefficients for the active sensors at time step $k = 3$ for the moving source example.

For the stationary source with added noise, the comparison between the mean value of the desired response $\mathbf{f}$ given in continuous k time steps and the mean value of the designed response $\mathbf{w}^H\mathbf{A}$ obtained by the first step of the proposed two-step method is shown in Figure 4. The mean value is calculated because there are small fluctuations at each time step due to the existing noise. The MSE between two responses is -70.7 dB. For the moving source with noise, the mean value of MSE is -71.4 dB, which is similar to the MSE result of moving source without noise.

Figures 5 and 6 show the DoA results for the stationary source and the moving source respectively. It can be observed that in both cases, the sources are still well tracked. However, the estimates are more noisy, and the SE between the estimated DoA ($(\mathbf{A}\mathbf{x}_k)^H\mathbf{A}$) and the target DoA ($\mathbf{y}_k^H\mathbf{A}$) becomes 0.55 degrees for the stationary source case and 0.31 degrees for the moving source case respectively.

V. Conclusion

A new method has been presented to exploit the joint sparsity in array design and source localisation. The method is operated in a two-step iterative process, of which the first step is to find the sensors to be used from the array and the second step is to perform source localisation with the LASSO algorithm with the selected sensors. The two steps are iterated in an alternating manner. The algorithm can start with a random guess of the DoA of the source when performing the optimisation for the sparse array, but eventually find the DoA of the source with the sparse reconstruction algorithm. The results evaluated for both stationary source and moving source show good performance of the proposed method. For the future work, we will extend the array from the narrowband to the wideband, as well as to a large scale.

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References

[1] M. R. Bai, J.-G. Ih, and J. Benesty, *Acoustic Array Systems: Theory, Implementation, and Application*. John Wiley & Sons, 2013.

[2] P. Händel, P. Stoica, and T. Söderström, “Capon method for doa estimation: accuracy and robustness aspects,” in *Proceedings of The IEEE Winter Workshop on Nonlinear Digital Signal Processing*. IEEE, 1993, pp. P–7.

[3] P. Totarong and A. El-Jaroudi, “A novel approach for robust high-resolution doa estimation,” in *Proceedings of The IEEE Sixth SP Workshop on Statistical Signal and Array Processing*. IEEE, 1992, pp. 366–369.

[4] A. Hakam, R. M. Shubair, and E. Salahat, “Enhanced doa estimation algorithms using mvdr and music,” in *2013 International Conference on Current Trends in Information Technology (CTIT)*. IEEE, 2013, pp. 172–176.

[5] P. Knee, “Sparse representations for radar with matlab® examples,” *Synthesis Lectures on Algorithms and Software in Engineering*, vol. 4, no. 1, pp. 1–85, 2012.

[6] D. L. Donoho, “Compressed sensing,” *IEEE Transactions on Information Theory*, vol. 52, no. 4, pp. 1289–1306, 2006.

[7] D. Malioutov, M. Çetin, and A. S. Willsky, “A sparse signal reconstruction perspective for source localization with sensor arrays,” *IEEE Transactions on Signal Processing*, vol. 53, no. 8, pp. 3010–3022, 2005.

[8] C. F. Mecklenbrauker, P. Gerstoft, A. Panahi, and M. Viberg, “Sequential bayesian sparse signal reconstruction using array data,” *IEEE Transactions on Signal Processing*, vol. 61, no. 24, pp. 6344–6354, 2013.

[9] E. Zochmann, P. Gerstoft, and C. F. Mecklenbrauker, “Density evolution of sparse source signals,” in *2015 3rd International Workshop on Compressed Sensing Theory and its Applications to Radar, Sonar and Remote Sensing (CoSeRa)*. IEEE, 2015, pp. 124–128.

[10] L. Harry and V. Trees, “Optimum array processing part iv of detection, estimation, and modulation theory,” *New York: John Wiler & Sons*, 2002.

[11] P. Jarske, T. Saramäki, S. K. Mitra, and Y. Neuvo, “On properties and design of nonuniformly spaced linear arrays [antennas],” *IEEE Transactions on Acoustics, Speech and Signal Processing*, vol. 36, no. 3, pp. 372–380, 1988.

[12] J. K. Schindler, “Compressive sensing for sparse arrays,” in *2013 IEEE International Symposium on Phased Array Systems & Technology*. IEEE, 2013, pp. 240–245.

[13] M. B. Hawes and W. Liu, “Robust sparse antenna array design via compressive sensing,” in *2013 18th International Conference on Digital Signal Processing (DSP)*. IEEE, 2013, pp. 1–5.

[14] N. Hu, Z. Ye, X. Xu, and M. Bao, “Doa estimation for sparse array via sparse signal reconstruction,” *IEEE Transactions on Aerospace and Electronic Systems*, vol. 49, no. 2, pp. 760–773, 2013.

[15] M. Barnard and W. Wang, “Adaptive bayesian sparse representation for underwater acoustic signal de-noising,” in *Proc. 2nd IET International Conference on Intelligent Signal Processing*, 2015.

[16] M. Grant, S. Boyd, and Y. Ye, “Cvx: Matlab software for disciplined convex programming,” 2008.