Mathematics B (Elec Eng): revision exercises 1

1. Expand (i) $(2x-y)^4$, (ii) $\dfrac{1}{(1-x^3)^{1/3}}$, (iii) $\dfrac{1}{(8-x^3)^{1/3}}$

2. Find the Maclaurin series of (i) $\cos x \sin 2x$, (ii) $x\ln(1+x^3)$, (iii) $\sec x$, as far as the x^4 term

3. Use L'Hopital's rule to find the limits

$$\lim_{x\to 1}\frac{e^{x-1}-1}{\ln x} \qquad \lim_{x\to 0}\frac{8x^2}{\cos x - 1} \qquad \lim_{\theta\to\pi/2}\frac{1-\sin\theta}{1+\cos 2\theta} \qquad \lim_{x\to\pi}\frac{\sin^2 x}{1+\cos x}$$

4. Find the exact values of (i) $\sinh(\ln 3)$, (ii) $\cosh(-\ln 2)$, (iii) $\tanh(2\ln 5)$.

5. If $\sinh x = -2$ find the values of $\cosh x$, $\tanh x$, $\operatorname{sech} x$, $\operatorname{cosech} x$ and $\coth x$.

6. Show that $\cosh \frac{1}{2}x = \sqrt{\frac{1}{2}(1+\cosh x)}$.

7. Evaluate

$$\text{(i)}\ \int \frac{dx}{\sqrt{1+2x^2}}, \qquad \text{(ii)}\ \int \frac{dx}{\sqrt{(x-1)(x+9)}}, \qquad \text{(iii)}\ \int_1^e \frac{dx}{x\sqrt{1+(\ln x)^2}}$$

Answers:

1. (i) $(2x-y)^4 = 16x^4 - 32x^3y + 24x^2y^2 - 8xy^3 + y^4$
(ii) $\dfrac{1}{(1-x^3)^{1/3}} = 1 + \frac{1}{3}x^3 + \frac{2}{9}x^6 + \frac{14}{81}x^9 + \cdots$
(iii) $\dfrac{1}{(8-x^3)^{1/3}} = \frac{1}{2} + \frac{1}{48}x^3 + \frac{1}{576}x^6 + \frac{7}{41472}x^9 + \cdots$.
2. (i) $\cos x \sin 2x = 2x - \frac{7}{3}x^3 + \frac{61}{60}x^5 + \cdots$
(ii) $x\ln(1+x^3) = x^4 - \frac{1}{2}x^7 + \frac{1}{3}x^{10} + \cdots$
(iii) $\sec x = 1 + \frac{1}{2}x^2 + \frac{5}{24}x^4 + \cdots$
3. 1, -16, $\frac{1}{4}$, 2
4. (i) $\frac{4}{3}$, (ii) $\frac{5}{4}$, (iii) $\frac{312}{313}$.
5. $\cosh x = \sqrt{5}$, $\tanh x = -2/\sqrt{5}$, $\operatorname{sech} x = 1/\sqrt{5}$, $\operatorname{cosech} x = -1/2$, $\coth x = -\sqrt{5}/2$.
7. (i) $\frac{1}{\sqrt{2}}\sinh^{-1}(\sqrt{2}x)$, (ii) $\cosh^{-1}((x+4)/5)$, (iii) $\sinh^{-1}(1)$.

Potentially useful information from formulae booklet

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots$$

(finite series if n is a positive integer or zero. If not, infinite series convergent when $|x| < 1$).

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$
$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots \qquad -1 < x \le 1$$
$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$$
$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$$
$$\arctan x = \tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots \qquad -1 \le x \le 1$$

$$\frac{d}{dx}\sinh^{-1}\left(\frac{x}{a}\right) = \frac{1}{\sqrt{a^2+x^2}}, \qquad \frac{d}{dx}\cosh^{-1}\left(\frac{x}{a}\right) = \frac{1}{\sqrt{x^2-a^2}}, \qquad \frac{d}{dx}\tanh^{-1}\left(\frac{x}{a}\right) = \frac{a}{a^2-x^2}$$

Note: the formulae booklet may use the notation arcsinh *instead of* $\sinh^{-1}$*; similarly for the other trigonometric and hyperbolic functions.*