UNIVERSITY OF SURREY©

Faculty of Engineering and Physical Sciences

Undergraduate Programmes in Electronic Engineering

Level 1

Module EEE1018: Mathematics

Time allowed: 2 hours | Spring Semester 2011

Answer all questions. All working must be shown.

The marks for each question are shown in brackets. You should note that some questions carry more marks than others.

You may use the **Tables of Constants, Formulae and Transforms**. Approved calculators may be used.

1.

(i) Find the binomial expansion of $\dfrac{1}{(1-x^2)^{\frac{1}{3}}}$ up to and including the term in x^6. [5]

(ii) Use the Maclaurin expansion of $\sin x$ (which you may quote from the booklet) to find the Maclaurin expansions of $\sin 3x$ and $\sin^3 x$, in each case up to and including the x^7 term. [For $\sin^3 x$ you may find the formula $\sin^3 x = \frac{1}{4}(3\sin x - \sin 3x)$ helpful.] [5]

Using these expansions (and no other method), find

$$\lim_{x\to 0} \frac{\sin^3 x - x^3}{x^5}$$ [3]

2. Using L'Hopital's rule or otherwise, find the limits

(i) $\lim_{x\to 2} \dfrac{x^5 - 32}{x - 2}$ (ii) $\lim_{x\to 0} \dfrac{\cos 2x - 1}{\cos x - 1}$ (iii) $\lim_{x\to\infty} x \sinh(5/x)$ [hint: let $y = 1/x$]

[3, 4, 4]

3.

(i) If $y = \sinh^{-1} x$ show that $e^y - e^{-y} = 2x$ and, by solving this equation for y, deduce that

$$\sinh^{-1} x = \ln(x + \sqrt{x^2+1}).$$ [5]

Hence show that if $\sinh y = \tan x$ then

$$y = \ln(\sec x + \tan x).$$ [3]

(ii) By applying Osborn's rule to the formula for $\sin^3 x$ in Question 1(ii), obtain a similar looking formula for $\sinh^3 x$ and hence show that

$$\int_0^1 \sinh^3 x \, dx = \tfrac{2}{3} - \tfrac{3}{8}(e + e^{-1}) + \tfrac{1}{24}(e^3 + e^{-3}).$$ [5]

4. Solve the following differential equations, expressing the solution in the form $y = f(x)$:

(i) $\dfrac{dy}{dx} = 2x\, e^{-3y}$ [4]

(ii) $\dfrac{dy}{dx} = \dfrac{y^3+1}{y^2}$ subject to $y(0) = 1$ [6]

(iii) $\dfrac{dy}{dx} = \cos x - \dfrac{y}{x}$ subject to $y(\pi) = 0$. [6]

[SEE NEXT PAGE]

5. Solve the differential equations:

(i) $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + 17y = 0$ subject to $y(0) = 1$ and $y'(0) = -1$ [7]

(ii) $\dfrac{d^2y}{dx^2} - 4\dfrac{dy}{dx} + 3y = 2x^2 + 3x - 1$ [8]

6. Sketch the graphs of the left and right hand sides of the equation $3\sin 2x = x^2 - 1$ and show that the equation must have a root between 1 and $\pi/2$. Show that the application of the Newton Raphson method to this equation leads to the recursion formula

$$x_{n+1} = x_n - \left(\frac{3\sin 2x_n + 1 - x_n^2}{6\cos 2x_n - 2x_n} \right).$$

With an initial guess of $x_0 = 1.2$, find an approximation to the root accurate to five decimal places. [10]

7.

(i) Evaluate $\displaystyle\int_0^2 \int_0^y (x^3 + xy^2)\, dx\, dy$. [5]

(ii) By converting to polar coordinates r and θ defined by $x = r\cos\theta$, $y = r\sin\theta$, evaluate

$$\iint_D \frac{x^2}{\sqrt{x^2 + y^2}}\, dA$$

where D is the region between the two circles $x^2 + y^2 = 4$ and $x^2 + y^2 = 9$. [You may quote that the Jacobian of the transformation is r]. [7]

8. A function $f(t)$ is 2π periodic and is given for $-\pi < t < \pi$ by $f(t) = 2t^2 + 1$.

(i) Draw the graph of $f(t)$ for $-3\pi < t < 3\pi$. [2]

(ii) Show that the Fourier series of $f(t)$ is

$$1 + \frac{2\pi^2}{3} + \sum_{n=1}^{\infty} \frac{8(-1)^n}{n^2} \cos nt.$$

[8]

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