UNIVERSITY OF SURREY©

Faculty of Engineering and Physical Sciences

Undergraduate Programmes in Electronic Engineering

Level 1

Module EEE1018: Mathematics

Time allowed: 2 hours

Spring Semester 2009

Answer all questions. All working must be shown.

The marks for each question are shown in brackets. You should note that some questions carry more marks than others.

You may use the **Tables of Constants, Formulae and Transforms**. Approved calculators may be used.

1.

(i) Find the binomial expansion of $(1+3x)^{1/3}$ up to and including the term in x^3. [5]

(ii) Use the expansions of e^x and $\cos x$ (which you may quote from the booklet) to find the Maclaurin expansion of $e^{-x^2}\cos 2x$ up to and including the term in x^4. [7]

2. Using L'Hopital's rule or otherwise, find the limits

(i) $\lim_{x\to 3} \frac{x^2-9}{x-3}$ (ii) $\lim_{x\to\pi/2} \frac{\cos x}{\pi-2x}$ (iii) $\lim_{x\to 1} \frac{3x^4-4x^3+1}{(x-1)\sin \pi x}$ [2, 4, 5]

3.

(i) If $y = a\cosh(x/a)+c$, where a and c are constants, show that

$$\frac{d^2y}{dx^2} = \frac{1}{a}\sqrt{1+\left(\frac{dy}{dx}\right)^2}.$$ [5]

(ii) Find

$$\int \cosh^2 3x \, dx$$ [4]

and

$$\int_2^3 \frac{dx}{\sqrt{x^2-4}}$$ [5]

4. Solve the following differential equations, expressing the solution in the form $y = f(x)$:

(i) $2xy^3 + \frac{dy}{dx} = 0$ [4]

(ii) $\frac{dy}{dx} = \frac{xy}{x+1}$ subject to $y(3) = 1$ [6]

(iii) $\frac{dy}{dx} - 5y - 5x = 0$ [5]

[SEE NEXT PAGE]

5. Solve the differential equations:

(i) $\dfrac{d^2y}{dx^2} + 5\dfrac{dy}{dx} - 14y = 0.$ [4]

(ii) $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + 2y = 10\sin 2x$ subject to $y(0) = 0$ and $y'(0) = -6$. [10]

6. Sketch the graphs of the left and right hand sides of the equation $2\cos 2x = 1 + 2x^3$, and show that the equation must have a root between 0 and $\pi/4$. Show that the application of the Newton Raphson method to this equation leads to the recursion formula

$$x_{n+1} = x_n - \left(\frac{1 + 2x_n^3 - 2\cos 2x_n}{6x_n^2 + 4\sin 2x_n}\right).$$

Making sure your calculator is in radian mode, find an approximation to the root accurate to four decimal places, using an initial guess of $x_0 = 0.5$. [11]

7.

(i) Evaluate $\displaystyle\int_1^2 \int_0^{\ln y} e^{-x}\, dx\, dy.$ [6]

(ii) By converting to polar coordinates r and θ defined by $x = r\cos\theta$, $y = r\sin\theta$, show that

$$\iint_D \sin\sqrt{x^2 + y^2}\, dA = 2\pi(\sin 2 - \sin 1 + \cos 1 - 2\cos 2)$$

where D is the region enclosed between the two circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$. [You may quote that the Jacobian of the transformation is r]. [7]

8. The complex form of the Fourier series of a T-periodic function $f(t)$ is

$$\sum_{n=-\infty}^{\infty} c_n\, e^{2jn\pi t/T}$$

where

$$c_n = \frac{1}{T}\int_0^T f(t)e^{-2jn\pi t/T}\, dt \qquad \text{for } n = 0, \pm1, \pm2, \pm3, \ldots.$$

Find c_n for the case when

$$f(t) = \begin{cases} 0 & 0 < t < 1 \\ 1 & 1 < t < 3 \end{cases}$$

with $f(t + 3) = f(t)$ for all t, and use your expression to show that

$$c_1 = \frac{1}{4\pi}(-\sqrt{3} + 3j).$$ [10]

Internal Examiner: S.A. GOURLEY