UNIVERSITY OF SURREY©

Faculty of Engineering and Physical Sciences

Undergraduate Programmes in Engineering

Level 1

ENG1002 Mathematics 1b

Time allowed: 2 hours

Spring Semester 2008

Answer all questions. All working must be shown. Approved calculators may be used. *The marks for each question are shown in brackets; you should note that some questions carry more marks than others.*

1. Solve the following differential equations for y, giving the answer in the form $y = f(x)$:

(i) $\dfrac{dy}{dx} = y\cos x$, [3]

(ii) $\dfrac{dy}{dx} = \dfrac{1-2x}{y}$ subject to $y(1) = -2$, [5]

(iii) $\dfrac{dy}{dx} + \dfrac{2}{x+1}\,y - 2 = 0$. [5]

2. Clean water enters a lake from a river at a constant rate r, and pollutant enters the lake from a factory at a constant rate p. The polluted water leaves the lake by another river such that the total volume V of the lake remains constant. Assume that the concentration $c(t)$ of pollutant in the lake at time t satisfies

$$\frac{dc}{dt} + \left(\frac{p+r}{V}\right)c = \frac{p}{V}$$

Solve this differential equation subject to $c(0) = 0$. What does $c(t)$ tend to as $t \to \infty$? [7]

3. Solve the following differential equations for y:

(i) $\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} - 12y = 0$ subject to $y(0) = 1$ and $y'(0) = -2$ [6]

(ii) $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + 2y = x^2 - x$ [7]

4. Find the partial derivatives $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ of each of the following:

(i) $z = 10xy - 7xy^2 - x^3y + 3x - 6y$ [4]

(ii) $z = xy + \dfrac{e^x}{y+1}$ [4]

5. A closed rectangular box is to be constructed. The material for the bottom costs 60 pence per square metre and the material for the sides and top costs 20 pence per square metre. The box is to have a volume of 6 cubic metres. If x and y are the length and breadth of the base, show that the cost of the box in pence is given by

$$C(x,y) = 80\left(xy + \frac{3}{x} + \frac{3}{y}\right)$$

Hence find the dimensions of the box that minimise the cost, giving each measurement to three decimal places. Confirm that you do indeed have a minimum. [12]

6. Write the system of simultaneous equations

$$\begin{aligned} 5x - y + 2z &= -1 \\ 2x + 4y + z &= 6 \\ x + 3y - 3z &= 8 \end{aligned}$$

in matrix form [2 marks]. Solve the system and check that the values you have found fit all three equations. [2 marks for each correct value, provided you have shown the working]. [8]

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7. Let A be the matrix

$$A = \begin{pmatrix} 2 & 3 & -1 \\ 2 & 2 & 1 \\ 1 & 2 & 0 \end{pmatrix}$$

Find A^2 and A^3 and show that $A^3 - 4A^2 - 3A + 3I$ is the zero matrix. [7]

8. (i) Find the rank of the matrix

$$\begin{pmatrix} 7 & -2 & 1 & -2 \\ 0 & 2 & 6 & 3 \\ 7 & 2 & 13 & 4 \\ 7 & 0 & 7 & 1 \end{pmatrix}$$ [6]

(ii) Show that, for any values of α, β and γ,

$$\begin{vmatrix} \sin^2\alpha & \sin^2\beta & \sin^2\gamma \\ \cos^2\alpha & \cos^2\beta & \cos^2\gamma \\ 1 & 1 & 1 \end{vmatrix} = 0.$$ [6]

9. Evaluate the following double integrals:

(i) $\int_0^4 \int_0^5 (x^2 y - 2xy)\, dy\, dx$ [5]

(ii) $\iint_D \cos\left(\frac{\pi x^5}{64}\right) dA$ where D is the region enclosed between the curve $y = x^4$, the x-axis and the line $x = 2$. [Hint: integrate in the y direction first]. Your solution should include a clear sketch of the region D. [8]

10. A thin plate of constant density occupies the region bounded by the x-axis and the curve $y = \sin x$, $0 \leq x \leq \pi$. Sketch this region. Taking advantage of any symmetries, find the coordinates of the centre of mass of the plate. [7]

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