

Learning Non-linear Models of Shape and Motion

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By

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Abstract

Deformable models have been an active area of research in computer vision for a number of years. Their ability to model non-rigid objects through the combination of geometry and physics has proven a valuable tool in image processing. More recently a class of deformable objects known as Point Distribution Models or Eigen Models have been introduced. These statistical models of deformation overcome some of the shortfalls of earlier deformable models by learning what is 'allowable' deformation, for an object class, from a training set of examples. This semi-automated learning procedure provides a more generic approach to object recognition, tracking and classification. Their strength lies in their simplicity and speed of operation, allowing the robust ability to model complex deformations in cluttered environments. However, the automated construction of such models leads to a breakdown of the fundamental assumptions upon which they are based. Primarily, that the underlying mathematical model is linear in nature. Furthermore, as more complex objects are considered, these assumptions fail completely and what is produced is an unreliable model.

This work addresses these problems and presents novel techniques for the automated construction and application of non-linear deformable models, which retain the speed, and simplicity of the linear Point Distribution Model. It is further shown how these non-linear models can be augmented with probabilistic temporal constraints, which are essential in object tracking and classification.

This work presents, in essence, three developments to the field. Firstly, a piecewise linear approach to modelling non-linearity is proposed and results demonstrated that show its accuracy in modelling both low and high dimensional datasets with heavy non-linearity. The technique is then extended to the automated construction of models. Secondly, it is shown how the piecewise approach can be augmented with temporal constraints and used in both model prediction, animation and for the support of multiple hypotheses during tracking. It is further shown how these temporal models can be extended to incorporate

information from other sources, providing more reliable tracking in the absence of complete training data. Thirdly, it is shown how elements can be combined statistically and used to infer information about an object from its shape alone. Using human motion capture as an example, it is demonstrated that models can be assembled which allow 3D structural information about body pose and motion to be inferred from a monoscopic image sequence using only natural features of the body as markers.

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Elements from this manuscript have appeared in, or are about to appear in the following publications.

Non-linear Statistical Models for the 3D Reconstruction of Human Pose and Motion from Monocular Image Sequences. , R. Bowden, T. A. Mitchell, M. Sarhadi, To appear in Image and Vision Computing.

Non-linear Point Distribution Models, R. Bowden, In CVonline: On-Line Compendium of Computer Vision [Online]. R. Fisher (ed). Section 11.3.1.2 , Oct 98.

Reconstructing 3D Pose and Motion from a Single Camera View, R. Bowden, T. A. Mitchell, M. Sarhadi, In Proc. BMVC, John N. Carter & Mark S. Nixon Eds, Uni of Southampton, Vol 2, pp , Southampton, Sept 1998.

Cluster Based non-linear Principal Component Analysis, R. Bowden, T. A. Mitchell, M. Sahardi, IEE Electronics Letters, 23rd Oct 1997, 33(22), pp1858-1859.

Real-time Dynamic Deformable Meshes for Volumetric Segmentation and Visualisation, R. Bowden, T. A. Mitchell, and M. Sahardi. In Proc. BMVC, Adrian F. Clark Ed, Vol 1, pp 310-319, Essex, UK, Sept 1997.

Virtual Datagloves: Interacting with Virtual Environments Through Computer Vision, R. Bowden, A. J. Heap and C. Hart, In Proc. 3rd UK VR-Sig Conference, DeMontfort University, Chris Hand Ed, Leicester, UK, July 1996.

Some elements of this work are similar in nature and content to the work of A. J. Heap. However, this work was done concurrently and in isolation as demonstrated by the publication, Cluster Based non-linear Principal Component Analysis, which was submitted to IEE Electronics Letters on 2nd Sept 1997 and predates the work of the other author.

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List of Abbreviations

ACM	Active Contour Model
ARM	Active Region Model
ASL	American Sign Language
ASM	Active Shape Model
BSL	British Sign Language
CBNLPDM	Cluster Based Non-linear Point Distribution Model
CSSPDM	Constrained Shape Space Point Distribution Model
CCD	Charge Coupled Device
CONDENSATION	Conditional Density Propagation
FCM	Fuzzy C Means (k-means) Clustering Algorithm
HSV	Hue Saturation Value
HSB	Hue Saturation Brightness
HLS	Hue Lightness Saturation
HSL	Hue Saturation Luminosity
HVC	Hue Value Chroma
$I_{x,y}$	Intensity of pixel at point (x,y)
ISL	International Sign Language
MRI	Magnetic Resonance Imaging
NLPDM	Non-linear Point Distribution Model
PCA	Principal Component Analysis
PDF	Probability Density Function
PDM	Point Distribution Model
RGB	Red Green Blue
ROI	Region of Interest
σ	Standard Deviation
Voxel	Volumetric Element