

# Time-frequency-modulation representation of stochastic signals

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# Introduction

- Modulated noise processes:
  - biomedical signals, e.g., with heartbeat/breathing
  - chemical plant affected by daily temperature variations
  - radiological or electromagnetic noise from machinery
  - passive sonar, turbines, road noise, voiced speech, etc.

$$x_n = w_n \left( 1 + \sum_{h=1}^H m_h \cos \left( \frac{2\pi k_h n}{N} + \phi_h \right) \right) \quad (1)$$

- Optimal demodulator [Van Trees, 2001]:
  - matched filter (inverse kernel)
  - envelope extraction (squaring or rectification; smoothing)

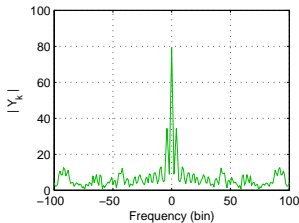
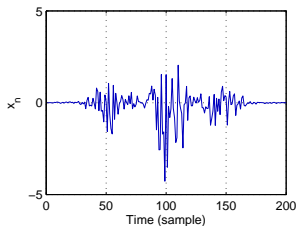
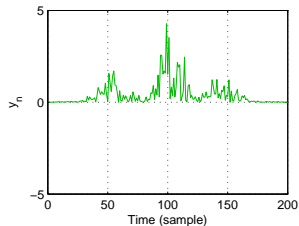
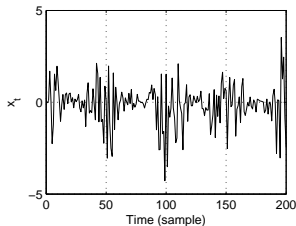
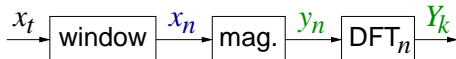
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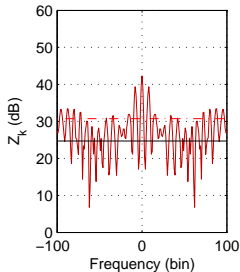
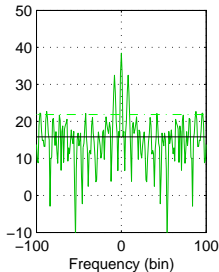
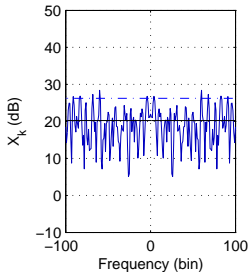
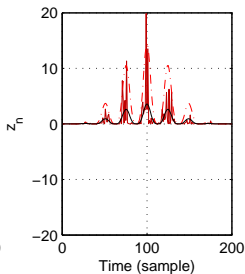
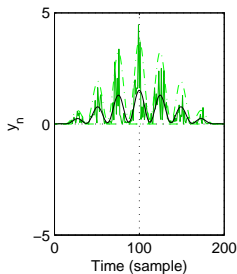
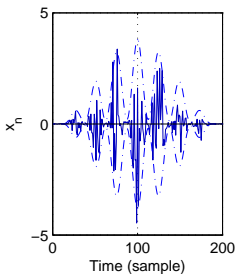
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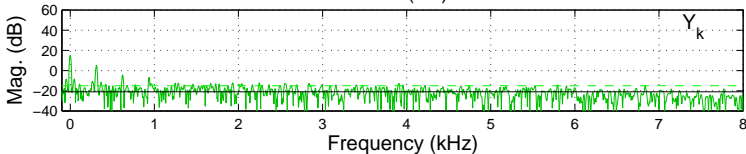
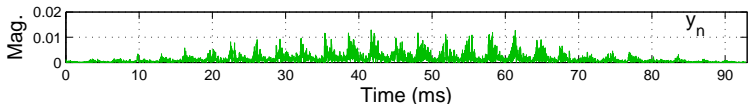
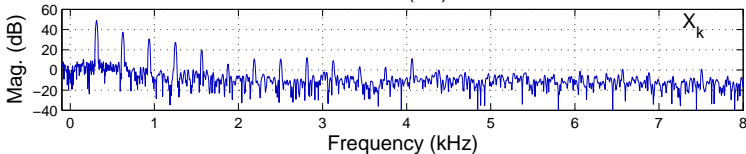
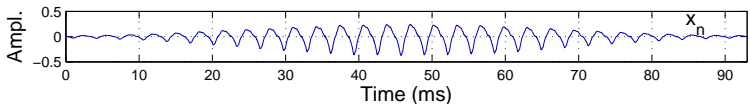
# Magnitude modulation spectrum



# Modulated white noise (Hann)



# MMS of voiced fricative speech



## Modulation index estimators

First-order statistic:

$$E\{Y_k\} = \sqrt{\frac{2}{\pi}} N \sigma \left[ \delta(k) + \frac{m}{2} \delta(k \mp k_1) e^{\pm j\phi_1} \right]$$

$$\dot{m}_1 = 2 \frac{|Y_{k_1}|}{|Y_0|} = 2 \left( \frac{Y_{k_1}^* Y_{k_1}}{Y_0^* Y_0} \right)^{\frac{1}{2}} \quad (2)$$

Second-order statistic:

$$E\{Y_k^* Y_k\} = \frac{2}{\pi} N^2 \sigma^2 \left[ \delta(k) + \frac{m^2}{4} \delta(k \mp k_1) e^{\pm j\phi_1} \right] + \theta$$

$$\ddot{m}_1 = 2 \left( \frac{Y_{k_1}^* Y_{k_1} - \theta^2}{Y_0^* Y_0 - \theta^2} \right)^{\frac{1}{2}} \quad (3)$$

## Simulation tests

- Gaussian white noise samples
  - 1200 files of 1 s duration, sampled at 44.1 kHz
- Sinusoidal modulator at  $f_1 = 150$  Hz, modulation index
  - $m = 0, 0.02, 0.05, 0.1, 0.15, 0.2, 0.3, 0.4, 0.5, 0.6, 0.8, 1$
- Variation of the analysis window size
  - $N = 1024, 2048, 4096, 8192, 16384$
  - 23 ms, 46 ms, 93 ms, 186 ms, 372 ms
- Variation of the modulating function
  - glide: A 0 Hz, B 20 Hz, C 60 Hz
  - jitter: A 0 %, B 0.5 %, C 1.5 %
  - multiple modulating frequencies,  $f_1$  and  $f_2$



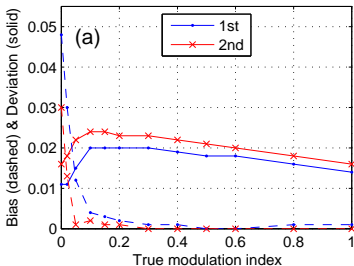
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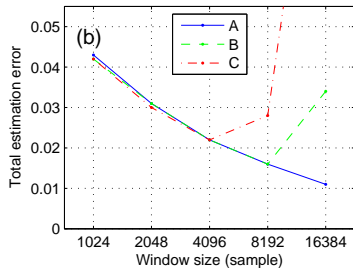
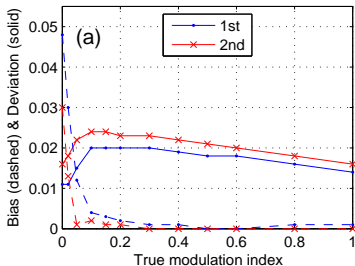
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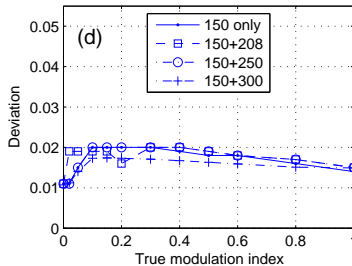
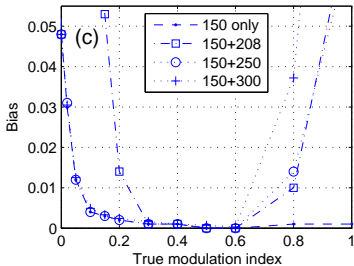
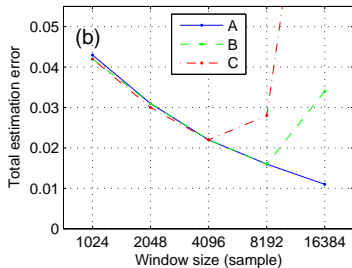
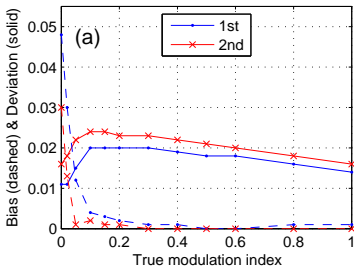
## Effect of modulation index



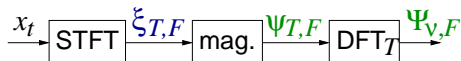
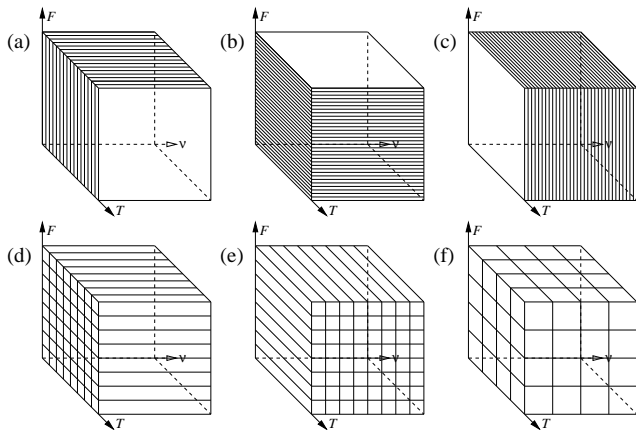
## Effect of window size



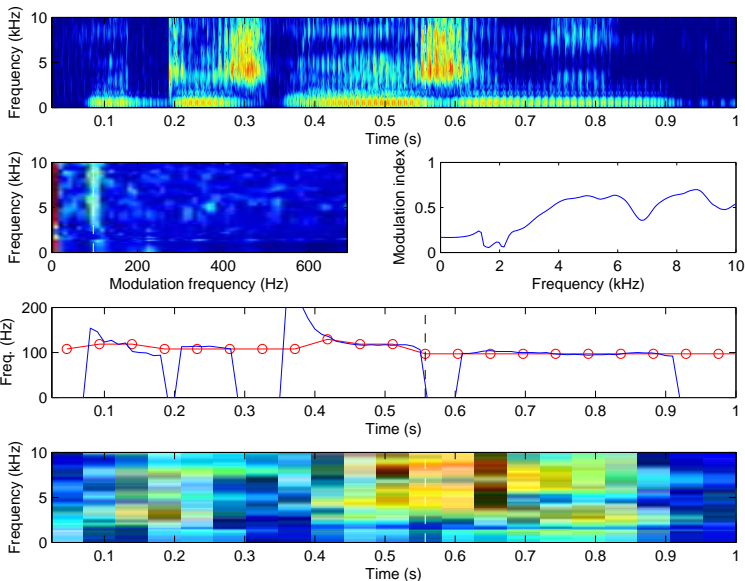
## Effect of two modulators



# Time, frequency and modulation



# TFM of voiced fricative speech



# Summary

- Conclusions
  - Magnitude modulation spectrum for  $m$  of AM noise
  - Estimators developed for real time-varying signals
  - Evaluation on synthetic test signals gave low error
  - Validated separate estimation of  $m$  for  $f_1$  and  $f_2$
  - Time-frequency-modulation proposed
- Future investigation
  - Theoretical bounds of first-order estimator
  - Applications to real stochastic signals

More?

<http://www.ee.surrey.ac.uk/personal/p.jackson/dsp07>

Thanks

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