

Define statistics:

Global statistics: $\Gamma = \{M, \Sigma\}$, grand correlation R^* , sample size N

Phone statistics: $\Lambda^\phi = \{\mu^\phi, \Sigma^\phi\}$, phone correlation R^ϕ , sample size N^ϕ

Model statistics $\Delta_k^\phi = \{m_k^\phi, S_k^\phi\}$, sample size N_k^ϕ

Threshold $\Theta = \{\theta_C, \theta_D\}$

Model initialisation:

level $k = 0$

$m_{k,i}^\phi = M_i$, $S_{k,i}^\phi = \Sigma_i$, $N_{k,i}^\phi = N$, for all articulators $i \in \{1..n\}$

Critical articulator list $C_k^\phi = \{\}$

$isConverge = 0$

while($k \leq n$ and $isConverge = 0$)

Identification divergence:

$J_{k,i}^\phi = \text{computeIdiv}(\Delta_{k,i}^\phi, \Lambda_i^\phi, N_{k,i}^\phi, N^\phi)$, for all articulators $i \in \{1..n\}$

Articulator with maximum divergence $j = \text{argmax}\{J_{k,1}^\phi, \dots, J_{k,n}^\phi\}$

C-step:

if($J_{k,j}^\phi > \theta_C$)

Go to the next level $k = k + 1$

$\Delta_k^\phi = \Delta_{k-1}^\phi$

$N_k^\phi = N_{k-1}^\phi$

Add critical articulator: $C_k^\phi \leftarrow \{C_{k-1}^\phi\} \cup \{j\}$

Set distribution: $m_{k,j}^\phi \leftarrow \mu_j^\phi$, $S_{k,j}^\phi \leftarrow \Sigma_j^\phi$

$N_{k,j}^\phi \leftarrow N^\phi$

D-step:

$[N_k^\phi, \Delta_k^\phi] = \text{updateDep}(\Delta_k^\phi, N_k^\phi, \Gamma, \Lambda^\phi, \Theta, C_k^\phi, R^*, R^\phi)$

else

Critical articulator list $\hat{C}^\phi = C_k^\phi$

Model distribution $\hat{m}^\phi = m_k^\phi$, $\hat{S}^\phi = S_k^\phi$

No. of critical articulators $K^\phi = k$.

$isConverge = 1$

end if

end while

function computeIdiv($\Delta_{k,i}^\phi, \Lambda_i^\phi, N_{k,i}^\phi, N^\phi$)

Incorporating standard error:

$S_1 = S_{k,i}^\phi + (S_{k,i}^\phi / N_{k,i}^\phi)$, $S_2 = \Sigma_i^\phi + (\Sigma_i^\phi / N^\phi)$

$J = \frac{1}{2} (tr(S_1 - S_2)(S_2^{-1} - S_1^{-1}) + tr(S_1^{-1} + S_2^{-1})(m_{k,i}^\phi - \mu_i^\phi)(m_{k,i}^\phi - \mu_i^\phi)')$

return J

function updateDep($\Delta_k^\phi, N_k^\phi, \Gamma, \Lambda^\phi, \Theta, C, R^*, R^\phi$)

Collate critical grand statistics: $M_{\{C\}}$, $\Sigma_{\{C\}\{C\}}$ (from Σ and R^*)

Collate critical phone statistics: $\mu_{\{C\}}^\phi$ and covariance matrix $\Sigma_{\{C\}\{C\}}^\phi$ (from Σ^ϕ and R^ϕ)

for $i \in \{1..n\} - \{C\}$

$J_{k,i}^\phi = \text{computeIdiv}(\Delta_{k,i}^\phi, \Lambda_i^\phi, N_{k,i}^\phi, N^\phi)$

if($J_{k,i}^\phi > \theta_D$)

Dependent covariance : $\Sigma_{i\{C\}}$

update mean: $m_{k,i}^\phi \leftarrow M_i + \Sigma_{i\{C\}} \Sigma_{\{C\}\{C\}}^{-1} (\mu_{\{C\}}^\phi - M_{\{C\}})$

update variance: $S_{k,i}^\phi \leftarrow \Sigma_i + \Sigma_{i\{C\}} \Sigma_{\{C\}\{C\}}^{-1} (\Sigma_{\{C\}\{C\}}^\phi - \Sigma_{\{C\}\{C\}}) \Sigma_{\{C\}\{C\}}^{-1} \Sigma_{i\{C\}}'$

$N_{k,i}^\phi \leftarrow N^\phi$

end if

end for

return Δ_k^ϕ, N_k^ϕ