Advanced Signal Processing

TUTORIAL SHEET 5

1. Write down the definition of the parametric form of the Bhattacharya distance as a function of the feature extraction matrix A.

2. (a) Find the transformation which decorrelates the components of pattern vectors drawn from a distribution with covariance matrix
$$\Phi = \begin{bmatrix} 3.5 & 1.5 \\ 1.5 & 3.5 \end{bmatrix}$$

   (b) Two equiprobable classes with mean vectors $\mu_1 = [4, 2]^T$ and $\mu_2 = [-4, -2]^T$ respectively have an equal covariance matrix Φ given in part 2a. Determine which of the K-L axes generated by matrix Φ contains most first order discriminatory information.

3. Consider two equiprobable classes with mean vectors $\mu_1 = [0, 0, 0]^T$ and $\mu_2 = [2, -1, 0]^T$ and covariance matrices
$$\Sigma_1 = \frac{1}{5}\begin{bmatrix} 21 & -8 & 0 \\ -8 & 9 & 0 \\ 0 & 0 & 5 \end{bmatrix} \quad \Sigma_2 = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

   (a) Determine the average within class covariance matrix and denote it by Φ.

   (b) Find the transformation which decorrelates matrix Φ.

   (c) Determine which of the K-L axes generated by matrix Φ contains the most of the first order discriminatory information.

   (d) Let the feature extractor be defined by the K-L axes determined in 3c. How much of the first order discriminatory information will remain in the two dimensional space which is orthogonal to the the one dimensional feature space.