# Machine Intelligence

## TUTORIAL SHEET 4

1. Write down the definition of Mahalanobis distance and contrast it with Euclidean distance.

2. Consider two normally distributed classes with mean vectors

$$\underline{\mu_1} = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \\ -1 \end{bmatrix} \quad \underline{\mu_2} = \begin{bmatrix} 2 \\ 1.5 \\ -0.3 \\ 0 \\ 1 \end{bmatrix}$$

   and equal covariance matrix

$$\Sigma = \begin{bmatrix} 2 & 1 & 0 & 2 & 1 \\ 1 & 1 & 0 & 1 & -1 \\ 0 & 0 & 3 & 0 & -2 \\ 2 & 1 & 0 & 3 & 2 \\ 1 & -1 & -2 & 2 & 8 \end{bmatrix}$$

   (a) Given that the best feature pair, in the sense of the Mahalanobis distance, is composed of features $\{x_1, x_4\}$, apply one step of the sequential forward selection algorithm to find the best triplet.

   (b) Compare the feature set obtained in part 2a with the best triplet selected using the Euclidean distance.

   You may wish to note the following:

$$\begin{bmatrix} 2 & 1 & 2 \\ 1 & 1 & 1 \\ 2 & 1 & 3 \end{bmatrix}^{-1} = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 8 \end{bmatrix}^{-1} = \frac{1}{13} \begin{bmatrix} 20 & -14 & 1 \\ -14 & 15 & -2 \\ 1 & -2 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 2 \\ 0 & 3 & 0 \\ 2 & 0 & 3 \end{bmatrix}^{-1} = \frac{1}{6} \begin{bmatrix} 9 & 0 & -6 \\ 0 & 2 & 0 \\ -6 & 0 & 6 \end{bmatrix}$$

3. Consider two normally distributed classes with mean vectors

$$\underline{\mu_1} = \begin{bmatrix} 2 \\ 3 \\ 3 \\ -2 \end{bmatrix} \quad \underline{\mu_2} = \begin{bmatrix} -1 \\ 2 \\ 1 \\ -6 \end{bmatrix}$$

   and equal covariance matrix

$$\Sigma = \begin{bmatrix} 4 & 1 & -3 & 0 \\ 1 & 2 & -1 & 0 \\ -3 & -1 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

   (a) Using the *Branch and Bound Search Algorithm*, determine the best subset of two features in the sense of the Mahalanobis distance.

   (b) Find the best pair of features using the Euclidean distance and compare it with the result of Part 3a

You may wish to note the following:

$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} & 0 \\ \frac{1}{5} & \frac{2}{5} & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$$

$$\begin{bmatrix} 4 & -3 & 0 \\ -3 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & \frac{4}{3} & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$$

$$\begin{bmatrix} 4 & 1 & -3 \\ 1 & 2 & -1 \\ -3 & -1 & 3 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & \frac{3}{5} & \frac{1}{5} \\ 1 & \frac{1}{5} & \frac{7}{5} \end{bmatrix}$$

$$\begin{bmatrix} 4 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{2}{7} & -\frac{1}{7} & 0 \\ -\frac{1}{7} & \frac{4}{7} & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$$