Machine Intelligence

TUTORIAL SHEET 3

1. State the Bayes minimum error decision rule.
2. Derive the optimal separating surface between two normally distributed classes with distinct mean vectors μ_i and covariance matrices Σ_i.
3. Consider two normally distributed classes with mean vectors
$$\mu_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \mu_2 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$
and covariance matrices
$$\Sigma_1 = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \quad \Sigma_2 = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$$
Let $P(\omega_1) = 0.3$. Determine the class to which pattern $\mathbf{x}^T = [1.0, 0.0]$ will be assigned to by the rule of Question 2.
4. Consider two exponentially distributed classes defined respectively by
$$p(\mathbf{x}|\omega_1) = \begin{cases} Ae^{ax} & 0 \le x \le 2 \\ 0 & elsewhere \end{cases}$$
$$p(\mathbf{x}|\omega_2) = \begin{cases} Be^{b(x-1)} & 1 \le x \le 3 \\ 0 & elsewhere \end{cases}$$
   (a) Derive a parametric form of the Bayes minimum error decision rule for the above densities.
   (b) Express the values of the normalising constants A and B in terms of the parameters a and b respectively.
   (c) Given $a = -1$ and $b = 0.5$, find the decision threshold separating the two classes, assuming that the class a priori probabilities are equal.
   (d) By how much will the class probabilities change as a result of observing pattern $x = 1.5$, as compared with the class probabilities before making the observation.
5. State the Nearest Neighbour decision rule
6. Express the average error rate of the Nearest Neighbour rule in terms of the Bayes error rate.
7. Consider a two class pattern recognition problem with the class conditional probability densities $p(\mathbf{x}|\omega_i),\ i = 1, 2$ defined as
$$p(\mathbf{x}|\omega_1) = \begin{cases} 2(1-x) & 0 \le x \le 1 \\ 0 & elsewhere \end{cases}$$
and
$$p(\mathbf{x}|\omega_2) = \begin{cases} 0.5 & 0 \le x \le 2 \\ 0 & elsewhere \end{cases}$$
where x denotes a one dimensional pattern and ω_i the i-th class. Given that the a priori probability $P(\omega_2)$ of class ω_2 satisfies $P(\omega_2) = 2P(\omega_1)$,
   (a) derive the aposteriori probability function for each class,
   (b) determine the class membership of point $x = 0.6$ using the Bayes minimum error rule,
   (c) find the Nearest Neighbour rule error rate and compare with the Bayes error