1. The Bayes minimum error decision rule:

$$\overset{\text{posterior probability}}{P(w_j|x)} = \frac{\overset{\text{class prior}}{P(w_j)}\ \overset{\text{likelihood}}{p(x|w_j)}}{Z} \leftarrow \text{normalising term}$$

and $Z = \sum_i P(w_i) p(x|w_i)$

We choose the maximum posterior probability:

$$\text{class label} = \arg\max_j P(w_j|x)$$

$$= \arg\max_j \frac{1}{Z} P(w_j) P(x|w_j)$$

$$\propto \arg\max_j P(w_j) P(x|w_j) \quad \text{(removing } Z\text{)}$$

In a two-class case,

2. choose class $\begin{cases} 1 & \text{if } p(w_1)p(x|w_1) \geq p(w_2)p(x|w_2) \\ 2 & \text{otherwise} \end{cases}$

The above rule can be written as:

$$\frac{p(w_1)p(x|w_1)}{p(w_2)p(x|w_2)} \geq 1$$

Take the log:

$f(x) \equiv \log\left\{\frac{p(w_1)p(x|w_1)}{P(w_2)p(x|w_2)}\right\} \geq 0.$ Hence we obtain the corresponding discriminative function.

When $p(x|w_i) = \mathcal{N}(x|\mu_i, \Sigma_i)$,

$$= \frac{1}{(2\pi)^{N/2}|\Sigma_i|^{1/2}} \exp\left(-\frac{1}{2}(x-\mu_i)^T\Sigma_i^{-1}(x-\mu_i)\right),$$

$$f(x) = -\frac{1}{2}(x-\mu_1)^T\Sigma_1^{-1}(x-\mu_1) - \frac{1}{2}\log|\Sigma_1| + \log p(w_1)$$
$$+\frac{1}{2}(x-\mu_2)^T\Sigma_2^{-1}(x-\mu_2) + \frac{1}{2}\log|\Sigma_2| + \log p(w_2)$$

Choose class $\begin{cases} 1 & \text{if } f(x) \geq 0 \\ 2 & \text{otherwise} \end{cases}$