Machine Intelligence

80 ms 02

**TUTORIAL SHEET 2**

12–1pm

1. Sketch a one-dimensional gaussian p.d.f. with mean $\mu = 3$ and variance $\sigma^2 = 4$.
2. Write down an expression for a gaussian p.d.f. in a multidimensional space.
3. For the gaussian p.d.f.s with the following mean vector $\mu$ and covariance matrix $\Sigma$

   $a)\ \ \mu = \begin{bmatrix} 1 \\ 2 \end{bmatrix}\ \ \Sigma = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\qquad b)\ \ \mu = \begin{bmatrix} 1 \\ 2 \end{bmatrix}\ \ \Sigma = \begin{bmatrix} 1.5 & 0 \\ 0 & 0.5 \end{bmatrix}$

   $c)\ \ \mu = \begin{bmatrix} 1 \\ 2 \end{bmatrix}\ \ \Sigma = \begin{bmatrix} 1.5 & 0.5 \\ 0.5 & 1.5 \end{bmatrix}$

   draw the equidensity curves and sketch the p.d.f.
4. For the gaussian p.d.f.s in Problem 3, find the density function values at points $\mathbf{x}_1 = [0, 0]^T$ and $\mathbf{x}_2 = [1, 2]^T$
5. The parameters $\mu_1$, $\Sigma_1$ and $\mu_2$, $\Sigma_2$ of two gaussian class conditional p.d.f.s $p(\mathbf{x}|\omega_i), i = 1, 2$ are given respectively as follows:

   $$\mu_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}\ \ \Sigma_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\qquad \mu_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}\ \ \Sigma_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

   - Write down the expression for the mixture density $p(\mathbf{x})$
   - Compute the mixture density at point $\mathbf{x} = [1, 1]^T$, assuming that the a priori probability of class $\omega_1$ is given as $P(\omega_1) = 0.8$
6. Express the joint probability $p(\mathbf{x}, \omega_i)$ in terms of conditional probabilities. Use the Bayes formula for relating conditional probabilities to compute the probability $P(\omega_1|\mathbf{x})$ at $\mathbf{x} = [1, 1]^T$ for the distributions in Problem 5. What is the conceptual difference between $P(\omega_i|\mathbf{x})$ and the a priori probability $P(\omega_i)$. What is the numerical difference between $P(\omega_1|\mathbf{x})$ and $P(\omega_1)$ at $\mathbf{x} = [1, 1]^T$.
7. Find the value $\alpha$ for the p.d.f. in Fig. 1.

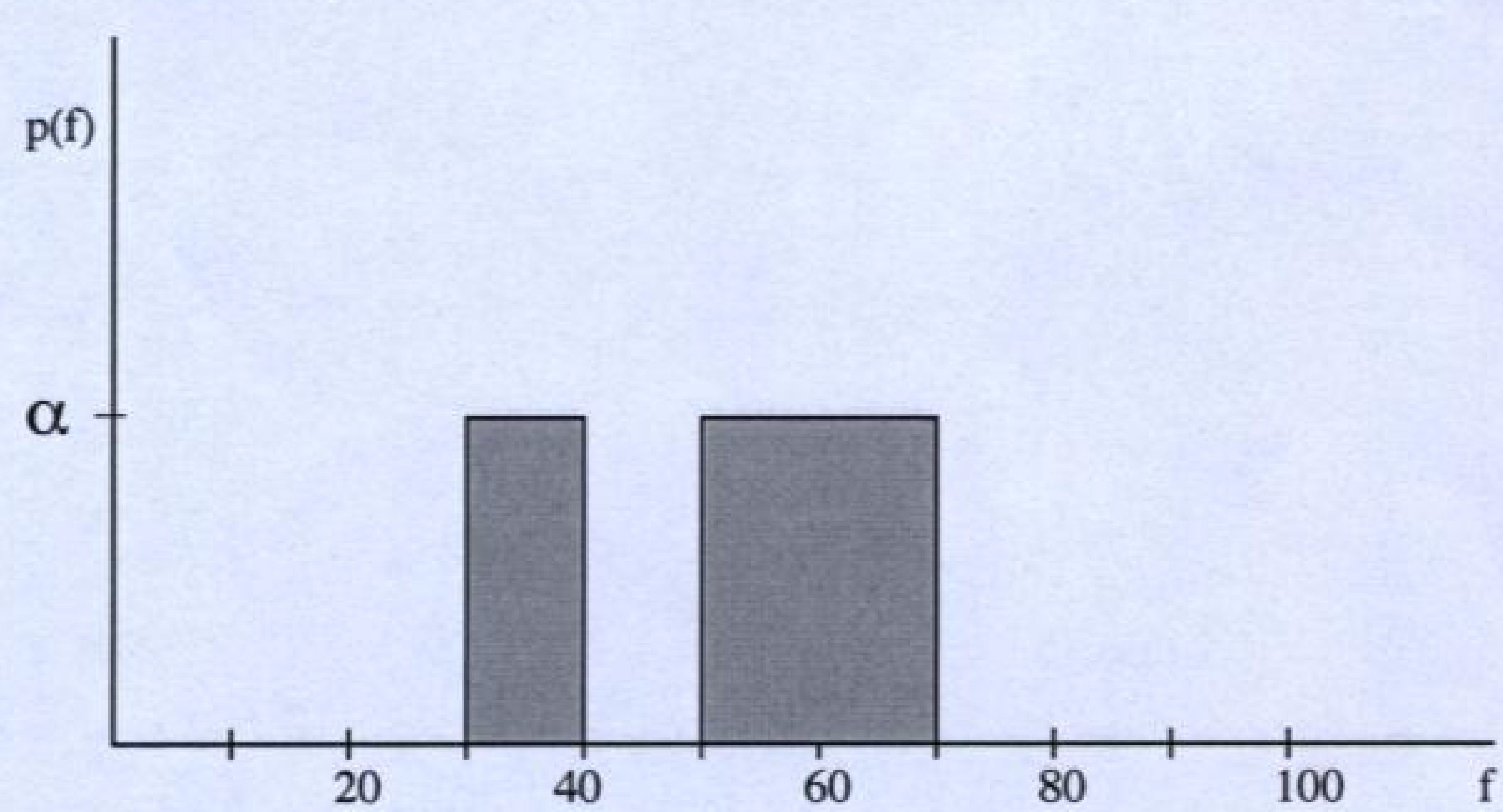


Figure 1: Probability density function