Machine Intelligence

TUTORIAL SHEET 1

1. Given vectors $\mathbf{z} = [1, 0, -3, 2, -1]^T$ and $\mathbf{y} = [2, 3, 1, 0, -1]^T$ find:

   i) $\mathbf{z} + \mathbf{y}$
   ii) $\mathbf{z} - \mathbf{y}$
   iii) $\mathbf{z}^T\mathbf{y}$
   iv) $\mathbf{z}\mathbf{y}^T$

2. Given $\Sigma = \begin{bmatrix} 4 & 1 & -3 \\ 1 & 2 & -1 \\ -3 & -1 & 3 \end{bmatrix}$, find:

   i) Σ^T
   ii) $det\Sigma$
   iii) $det\Sigma^T$
   iv) $tr\Sigma$
   v) Σ^{-1}
   vi) $\Sigma \times \Sigma$
   vii) $det(3\Sigma)$

3. Given $\mathbf{x} = [1, -1, 2]^T$ and $\Sigma = \begin{bmatrix} 4 & 1 & -3 \\ 1 & 2 & -1 \\ -3 & -1 & 3 \end{bmatrix}$, find:

   i) $\Sigma\mathbf{x}$
   ii) $\mathbf{x}^T\Sigma\mathbf{x}$
   iii) $\mathbf{x}^T\Sigma^{-1}\mathbf{x}$
   iv) $det\{\mathbf{x}\mathbf{x}^T\Sigma\}$
   v) $tr\{\mathbf{x}\mathbf{x}^T\Sigma\}$

4. Given $\Sigma_1 = \begin{bmatrix} 4 & 1 & -3 \\ 1 & 2 & -1 \\ -3 & -1 & 3 \end{bmatrix}$ and $\Sigma_2 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ find:

   i) $det\Sigma_2$
   ii) Σ_2^{-1}
   iii) $tr\Sigma_2$

   Show that:

   iv) $(\Sigma_1\Sigma_2)^{-1} = \Sigma_2^{-1}\Sigma_1^{-1}$
   v) $\Sigma_1\Sigma_1^{-1} = I$
   vi) Σ_2 is positive definite

5. What is the magnitude of vectors $\mathbf{z}$ and $\mathbf{y}$ given in Problem 1.

6. Find the eigenvalues λ_i and eigenvectors $\mathbf{u}_i$, $\quad i = 1, 2$ of matrix Σ

$$\Sigma = \begin{bmatrix} 1.5 & 0.5 \\ 0.5 & 1.5 \end{bmatrix}$$

   How many distinct eigenvalues and eigenvectors are there? Are the eigenvectors orthogonal? What is the magnitude of the eigenvectors. Normalise the eigenvectors so that they become orthonormal.

7. Show that the magnitudes of vector $\mathbf{x} = [1, -1]^T$ and $\mathbf{w} = \mathbf{U}^T\mathbf{x}$, where $\mathbf{U} = [\mathbf{u}_1, \mathbf{u}_2]$ is the matrix of normalised eigenvectors derived in Problem 6, are identical.

8. Find the Euclidean distance between vectors $\mathbf{z}$ and $\mathbf{y}$ given in Problem 1: