

PDEs: UN-ASSESSED HOMEWORK I

We advise you to solve ALL the questions below.

However, please hand in only the solutions of the starred questions at the end of our lectures on Thursday 6/3/2014.

These questions will be marked and returned to you with appropriate feedback.

Thought of the homework:

Partial differential equations are the basis of all physical theorems. In the theory of sound, liquids and solids, in the investigation of elasticity, in optics, everywhere partial differential equations formulate basic laws of nature which can be checked against experiments.

Bernhard Riemann

For each of the following linear partial differential equations, find its characteristic equations and then find its general solution. Hence find the solution satisfying the initial data on the given curve.

a*) $u_x + u_y + 2u = 0$.

Find a solution satisfying $u(x, y) = \sin(x)$ on the line $y = 0$.

b) $xu_x - (x \tan x)u_y + u + \frac{1}{x^2} = 0$.

Find the set of points where b) is not defined.

c*) $3yu_x - 2xu_y = 0$.

i) Find a solution satisfying $u(x, y) = x^2$ on the line $y = x$.

ii) Find a solution satisfying $u(x, y) = 2x$ on the ellipse $3y^2 + 2x^2 = 4$.

d) $yu_x + x^2u_y - xy = 0$.

Find a solution satisfying $u(x, y) = 4x$ on the curve $y = \frac{1}{3}x^{3/2}$.

e*) $xu_x + u_y - u = 0$.

Find a solution satisfying $u(x, y = 0) = x^2$.

f*) $yu_x + xyu_y + u = 0$.

Find a solution satisfying $u(x, y = \frac{3x^2}{2}) = x^2 e^{-\frac{\sqrt{2}}{x}}$.