

Operations Research for Computing, Spring 2010, Coursework 2

Please hand this in to the undergraduate office no later 0900 on Tuesday April 27 or if submitted electronically, to the lecturer, not later than 2359 on Monday April 26

Remember that this coursework must be *all your own work*.

1. A firm can manufacture four liquid speciality chemical products at its factory. Each product requires time on three different machines but production is limited by the time available on each of the machines. The firm wishes to maximise the profit made by manufacturing these products. The data is summarised below in hours per 100 litres and £ profit per 100 litres of product.

	Product				Availability
	1	2	3	4	
Machine 1 (hours)	1	4	2	3	100 hours per week
Machine 2 (hours)	3	1	5	3	150 hours per week
Machine 3 (hours)	2	3	1	3	120 hours per week
Profit	25	38	20	30	

- (a) Formulate this problem as a Linear Programming problem in standard form. [3]
- (b) Solve this problem using Excel Solver assuming that the firm can sell all the product it makes. [6]

[Include a copy of the worksheet that defines the problem as well as the corresponding Answer Report and Sensitivity Report for each part of this question. Marks will only be given if you use *only* the information contained in these worksheets in your answers]

- (c) From your solution, state the amount of each type of product that should be made in order to maximise profits. What is the corresponding profit that is made? [3]
- (d) Explain clearly why the optimal solution will always involve the firm's making none of at least one product. [3]
- (e) The firm can increase the available hours for one of the machines. The costs of each additional hour are £1.50 per hour for machine 1, £4 per hour for machine 2 and £5 per hour for machine 3. On which machine should they increase the available time, by how much and how much extra profit would the firm make as a result? [5]
- (f) The firm can further process product 4 so that it can be sold at a profit of £40 per litre by acquiring machine 4 which is available for 105 hours per week at a cost of £12 per hour. Is this profitable for the firm and what should the new production schedule be? [5]
- (g) The firm wants to maintain the optimal production schedule determined in part (b). However, it wants to increase its profit as much as it can in order to fund a pay rise for the employees. To test the market, the firm has decided that it will only increase the profit on one of the four products. For which of the four products should the profit be increased and by how much, if the current solution is to remain optimal? How much extra profit would the firm make in this case? [5]

- (h) Half the work force has gone on strike and so production at the factory has been affected. Now a maximum of 4,500 litres of products per week can be made. What should the new production schedule be and what is the corresponding profit? [5]
- (i) The company has the opportunity to manufacture product 5 which requires 3 hours on machine 1, 4 hours on machine 2 and 2 hours on machine 3. How much profit would need to be made per 100 litres of product 5 before it is profitable for the firm to produce it? [5]

Note that each of the scenarios described in this question is to be considered separately, not in combination.

2. A firm assembles motherboards which contain the Euler chip. The firm uses 120,000 Euler chips each year at a constant rate and the factory works continuously. The chips must be stored in a refrigerator so their holding cost is high - £0.25 per chip per month. The chip supplier imposes a £500 delivery charge on each order.
- (a) Determine the optimal order quantity, the time between orders and the annual cost of inventory if no shortages are allowed. [3]
- (b) The present policy is to order stock every two months, how much would be the annual savings by ordering the optimal quantity? [2]
- (c) The purchase price of the Euler chip is usually £4 per chip but the supplier is offering a quantity discount of 10% on orders of 10,000 or more. Should the company accept this discount and if it does what is the new optimal strategy and annual inventory cost. [10]
- (d) If there is a shortage of Euler chips in the factory at any time the firm is obliged to pay an additional £3 per Euler chip to obtain them from another source (i.e. the shortage cost is £3 per chip). If the firm is willing to accept planned shortages, what would be the new optimal strategy and annual inventory cost. What would be the maximum shortage allowed? How much of the time is there a shortage of chips? [5]

3. Consider the following transportation problem - five factories supply five warehouses, the costs are in £ per ton and the supply and demand are in tons.

	W_1	W_2	W_3	W_4	W_5	Supply
F_1	6	4	5	6	10	35
F_2	11	7	9	8	7	10
F_3	3	10	8	13	11	20
F_2	9	4	11	2	3	10
F_3	8	6	7	0	6	20
Demand	10	20	30	12	28	

- (a) Use the **north-west corner method** to find an initial BFS (**No marks will be given for using the least-cost method**). How many basic variables are there. Explain clearly why there are this number? [6]
- (b) Check the initial BFS for optimality and continue iterating the transportation algorithm until you have found an optimal solution. Show all your working. Explain carefully what is meant by a shadow cost (s_{ij}). Clearly state your solution. [15]
- (c) There has been a major fire at Factory 2 and the factory is closed. Find the new optimal solution **using the least-cost method**. How is the shortfall from the factory closure allocated? [7]
- (d) Due to a strike, Warehouse 4 is closed. The storage costs for excess production at factories 1 to 5 are £8, £8, £10, £12 and £10 respectively. Find the new optimal solution **using the least-cost method**. How is the excess production allocated. [7]
- (e) Set up the problem in Excel solver and verify the overall transportation costs you have found in parts (b), (c) and (d). (You will probably get a slightly different set of basic variables with the same overall transportation cost). **Include the answer report with your solutions.** [5]