

## Solutions to exercises on page 56

1. (a)

$$f(x) = \sum_{n=1}^{\infty} \frac{2}{n} (-1)^{n+1} \sin nx$$

(b)

$$f(x) = \frac{\pi^2}{3} + \sum_{n=1}^{\infty} \frac{4}{n^2} (-1)^n \cos nx$$

(c)

$$f(x) = \sum_{n=1}^{\infty} \frac{2}{\pi n} (1 - (-1)^n) \sin nx$$

(d)

$$f(x) = \frac{1}{2} - \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos(2n-1)x - \sum_{n=1}^{\infty} \frac{2(-1)^{n-1}}{\pi(2n-1)^2} \sin(2n-1)x + \sum_{n=1}^{\infty} \frac{(-1)^n}{2n} \sin 2nx$$

(e) See lecture notes.

2. (a)

$$f(x) = 1 + \sum_{n=1}^{\infty} \frac{4((-1)^n - 1)}{n^2 \pi^2} \cos \frac{n\pi x}{2}$$

(b)

$$f(x) = \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \left( \frac{2}{n^2 \pi^2} - \frac{1}{3} \right) \sin n\pi x$$

(c)

$$f(x) = \frac{1}{4} + \sum_{n=1}^{\infty} \frac{1 - (-1)^n}{n^2 \pi^2} \cos 2n\pi x$$

(d)

$$f(x) = 2 - \sum_{n=1}^{\infty} \frac{4 \cos^2(n\pi)}{4n^2 - 1} \cos 2n\pi x - \sum_{n=1}^{\infty} \frac{4 \cos n\pi \sin n\pi}{4n^2 - 1} \sin 2n\pi x$$

3. (a)

$$f(x) = \frac{\pi}{2} \sum_{n=1}^{\infty} \frac{2(-1)^n - 2}{\pi n^2} \cos nx$$

(b)

$$f(x) = \frac{2}{\pi n} (1 - (-1)^n) \sin nx$$