Answers to Exercises in MS100 Notes (Part B)

Page 34–35

1. (a) Amplitude = 3, period = $2\pi/3$.
   (b) This is $4\sin 6x$, so amplitude = 4, period = $\pi/3$.
   (c) Amplitude = 2, period = π.
   (d) This is $10\sin(x+\alpha)$ for some α so amplitude = 10, period = 2π.

2. $\cosh^2 x - \sinh^2 x = \frac{(e^x+e^{-x})^2}{4} - \frac{(e^x-e^{-x})^2}{4}$

   $= \frac{(e^{2x}+e^{-2x}+2)-(e^{2x}+e^{-2x}-2)}{4} = \frac{2+2}{4} = 1$

3. (a) $4\sinh^2 x = 1 + \sinh^2 x \Rightarrow 3\sinh^2 x = 1 \Rightarrow \sinh x = \pm\frac{1}{\sqrt{3}}$

   $x = \text{arsinh}\,(1/\sqrt{3}) = \frac{1}{2}\ln 3$, or $x = \text{arsinh}\,(-1/\sqrt{3}) = -\frac{1}{2}\ln 3$.

   (b) $7(e^x - e^{-x}) = 48 \Rightarrow 7e^{2x} - 48e^x - 7 = 0 \Rightarrow (7e^x + 1)(e^x - 7) = 0$

   $e^x > 0$ for all x, so $e^x = 7$ $\quad x = \ln 7$

4. For $x > 1$, $\text{arcosh}\, x = \ln\,(x + \sqrt{x^2-1})$

   Thus $\text{arcosh}\,(13/12) = \ln\,(13/12 + 5/12) = \ln\,(3/2)$ or $\ln 3 - \ln 2$

5. $\cosh 2x = 3 - 2\cosh x \Rightarrow 2\cosh^2 x - 1 = 3 - 2\cosh x$

   $\cosh^2 x + \cosh x - 2 = 0$ $\quad (\cosh x - 1)(\cosh x + 2) = 0$

   $\cosh x = 1$ or $\cosh x = -2$ $\quad x = 0$ $\quad$ Only point is (0, 1)

6. We know that $\cosh^2 x - \sinh^2 x \equiv 1$

   Dividing through by $\sinh^2 x$, we get $\coth^2 x - 1 \equiv \text{cosech}^2 x$, so $\coth^2 x - \text{cosech}^2 x \equiv 1$.

   $1 + \text{cosech}^2 x = 2\,\text{cosech}\, x \Rightarrow \text{cosech}^2 x - 2\,\text{cosech}\, x + 1 = 0$

   $(\text{cosech}\, x - 1)^2 = 0 \Rightarrow \text{cosech}\, x = 1 \Rightarrow \sinh x = 1$

   $e^x - e^{-x} = 2$ $\quad e^{2x} - 2e^x - 1 = 0$

   $e^x = (2 \pm \sqrt{8})/2 = 1 \pm \sqrt{2}$ $\quad x = \ln\,(1 + \sqrt{2})$

7. (a) $\lim_{x\to 0}(2x+3) = 3$, (b) $\lim_{x\to\infty} \frac{12 + \frac{6}{x}}{3 - \frac{4}{x}} = 4$, (c) $\lim_{x\to 3} \frac{1}{x+3} = \frac{1}{6}$.

8. $f(x) = \frac{80 + 60x^{0.3}}{8 + 15x^{0.3}}$.

   (i) As $x \to 0$, $f(x) \to 10$. (ii) As $x \to \infty$, $f(x) \to 4$.

9. (a) Crosses axes at (0, 0). Asymptotes $x = -1, y = 1$. Range $\mathbf{R} \setminus \{1\}$
   (b) Crosses axes at (0, 2/3), (–2/3, 0). Asymptotes $x = 3/2, y = -3/2$. Range $\mathbf{R} \setminus \{-3/2\}$
   (c) Crosses axes at (0, 2). Asymptotes $x = \pm 1, y = 0$. Minimum turning point at (0, 2). Even function. Range $\mathbf{R} \setminus [0, 2)$.
   (d) Crosses axes at (0, 0). Asymptotes $y = \pm 1$. Point of inflexion at (0, 0). Odd function. Range $\mathbf{R}$.

(e) Crosses axes at (0, 1), (–1, 0), (2, 0). Asymptotes $x = 1, x = --2, y = 1$. Range **R**

(f) Crosses axes at (–1.618, 0), (0.618, 0). Asymptotes $x = 1, y = x + 2$. Turning points at $(0, 1), (2, 5)$. Range $(-\infty, 1] \cup [5, \infty)$.

(g) Crosses axes at (0, 1/2). Asymptote $y = 1$. Minimum turning point at (0, 1/2). Even function. Range $[1/2, 1)$.

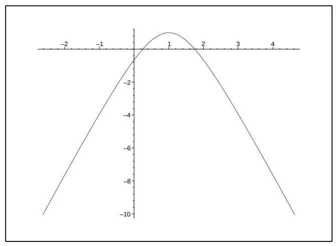
10. $\mathrm{f}(x) = \dfrac{x(2 + \sqrt{6x^2 + 4})}{2}$.

11. Circle is $(x - 3)^2 + (y + 4)^2 = 169$, so centre is $(3, -4)$ and radius = 13.

    64 + 64 – 48 + 64 – 144 = 0, so (8, 8) lies on C.

    Vector from (8, 8) to (3, –4) is (–5, –12), so B is (–2, –16).

12. Curve symmetric about $x = 1$:



When $y = 0$, $4 \cosh t = 5 \Rightarrow 2\mathrm{e}^t + 2\mathrm{e}^{-t} - 5 = 0$

$2\mathrm{e}^{2t} - 5\mathrm{e}^t + 2 = 0 \Rightarrow (2\mathrm{e}^t - 1)(\mathrm{e}^t - 2) \Rightarrow \quad t = -\ln 2$ or $\ln 2$

Then $x = 1 + \sinh t = 1 \pm \frac{3}{4}$. Points are $\left(\frac{1}{4}, 0\right)$ and $\left(\frac{7}{4}, 0\right)$.