MAT 1015 Techniques in Calculus I Autumn 2009

Coursework 3

Please hand in your solution to me in the 1000 lecture on Friday 18th December , clearly marked with **your name and that of your tutor**. You should show all your working as part of your answers. The questions with an asterisk (*) may be a little more challenging.

1. Find the first four non-zero terms and the radius of convergence of the Maclaurin Series for the following functions;

$$\text{(a)} \quad \tan^{-1} x \qquad \text{(b)} \quad \ln\left(\frac{1-x}{1+x}\right) \qquad \text{(c)} \quad \frac{e^x}{1+x}.$$

Hence evaluate $\displaystyle\int_0^{0.2} \frac{e^x}{1+x}\,dx$ accurate to 3 d.p.s.

2. Prove that

$$\lim_{x\to 0} x^2 \sin\left(\frac{3+x)}{x^4}\right) = 0.$$

3. (a) **Without** using l'Hôpital's rule find:

$$\text{(i)} \quad \lim_{x\to 5} \frac{3x^2-16x+5}{x^2-3x-10} \qquad \text{(ii)} \quad \lim_{x\to 0} \frac{17x^{-0.25}+8}{3x^{-0.33}+2} \qquad \text{(iii)} \quad \lim_{x\to\infty} \frac{3x^4-7x^2-1}{8x^4+4x^2+5}.$$

(b) **Using** l'Hôpital's rule find:

$$\text{(i)} \quad \lim_{x\to 1} \frac{\tan(x-1)}{x^2-1} \qquad \text{(ii)} \quad \lim_{x\to 0} \frac{10^x-e^x}{x} \qquad \text{(iii)} \quad \lim_{x\to 0} \frac{2\sin x-\sin 2x}{2e^x-2-2x-x^2}.$$

4. Find the area lying inside both of the circles $x^2+y^2=1$ and $(x-2)^2+y^2=4$.

5. Find the mean value of $y=\sqrt{4-x^2}$ over $[0,2]$.

6. Find the length of the curve $y^3=x^2$ from $x=-1$ to $x=1$.

7. Find the area of the surface produced by rotating the ellipse $x^2+4y^2=4$, around the x axis

8. Find the following integrals:

$$\text{(a)} \quad \int \frac{dx}{1+\sin x+\cos x} \qquad \text{(b)} \quad \int \left(x^2+2\,x+5\right)^{-1} dx$$

$$\text{(c)} \quad \int \frac{1}{\sqrt{x^2+2\,x+5}}dx \qquad \text{(d)} \quad \int \left(x^2+2\,x+5\right)^{-2} dx \qquad \text{(e)} \quad \int \frac{2x^3-5x^2+4x-4}{x^2-x}dx$$

9. Find the series representation of each of the following integrals, stating the range of values of x for which each series converges:

$$(a) \quad \int \frac{\sin x}{x} dx, \quad (b) \quad \int \frac{e^x - 1}{1 - x^2} dx.$$

10. By direct multiplication of the Maclaurin Series for e^x and e^y, show that $e^x e^y = e^{x+y}$.

11. Show that

$$\int f^{-1} x \, dx = x f^{-1}(x) - \int f(y) dy.$$

12. * Evaluate the following integrals:

$$(a) \quad \int_0^\infty (1 - x) e^{-x} dx \quad (b) \quad \int x \cos^{-1} x \, dx.$$

13. * Find the value of c for which the following integral converges and evaluate the integral at that value.

$$\int_0^\infty \left(\frac{1}{\sqrt{x^2 + 1}} - \frac{c}{x + 1} \right) dx.$$

14. * What is wrong with the following calculation;

$$\int_{-1}^{1} \frac{dx}{x^2} = -\frac{1}{x} \Big|_{-1}^{1} = -1 + \frac{1}{-1} = -2,$$

where did the error occur and why is -2 an unreasonable value for the integral?