

COM 1021 Mathematical Methods for Computing I Autumn 2009

COURSEWORK I

To be handed in no later than the 1100 lecture on Monday 23 November

While you may want to use Maple to check your answers, only questions marked with an asterisk (*) may be answered entirely using Maple. For all the other questions, marks will **only** be given if **all** your relevant working is shown.

This coursework counts for 15% of the marks for this module.

Your solutions must be all your own work.

Functions

- (a) Find the **range** of the function in each case:
(i) $y = x^2 + 6x + 1$ when $x \in \mathbb{R}$ (ii) $y = 2 - 3x^2$ when $x \in [-4, 2)$. [4]
- (b) Find the **domain** of the function in each case:
(i) $y = x^2 - x - 2$ (ii) $y = \sqrt{x^2 - x - 2}$ (iii) $y = \frac{1}{x^2 - x - 2}$. [5]
2. If $f(x) = 4x - 3$ and $g(x) = 5 - 2x$;
 - Find
(i) $f^{-1}(x)$ (ii) $g^{-1}(x)$ (iii) $f(g(x))$ (iv) $g(g(x - 1))$
 - Find
(i) $g(f(x)) - f(g(x))$ (ii) $f^{-1}(g^{-1}(x)) - g^{-1}(f^{-1}(x))$.
 - Find $f(f(f(x)))$, $f^{(4)}(x)$ and $f^{(5)}(x)$. Can you find a general expression for $f^{(n)}(x)$?
 - Suppose $F(x) = f(x^2)$ and $G(x) = g(x^2)$. Solve $F(G(x)) = 0$, expressing your answers in exact (i.e. surd) form. [27]
3. Find the inverse of
(i) $y = \ln(3x + 2)$ (ii) $y = \frac{1 + 3x}{3 - x}$ [4]

Graphs

4. Sketch the graphs of the following, clearly indicating the coordinates of the points at which the curve crosses the x and y axes and any local minima or maxima.
 - $2y + 3x - 4 = 0$ (b) $y = x^2 - 5x + 4$ (c) $x^2 - 6x + y^2 + 4y - 23 = 0$ [10]

TURN OVER THE PAGE

Equations and inequalities

5. The pressure P experienced by a diver under water is related to the diver's depth d by the equation

$$P = kd + c.$$

At the surface the pressure is 1 atmosphere and at a depth of 200 metres the pressure is 21 atmospheres.

- (a) Find c and k .
- (b) Find the pressure at 117 metres.
- (c) Find the depth at which the pressure is 39 atmospheres. [3]
6. Find all the solutions of the following equations
- (a) $3x^2 - 4x - 5 = 0$,
- (b) $6x^2 + x - 2 = 0$,
- (c) $x^3 - 5x^2 + 6x = 0$. [7]
7. (a) Solve the following inequalities and sketch the appropriate areas on a graph.
- (i) $(x - 3)^2 \leq 16$ (ii) $|x - 3| > 16$. [6]
- (b) Sketch the areas on the graph represented by
- (i) $3 < x < 8, -1 < y < 5$ (ii) $x + y \leq 5$. [6]

Trigonometry

8. (a) Find the distance between the points $(-2, -7)$ and $(5, -4)$
- (b) In a right angled triangle the hypotenuse is of length 12 and one of the angles is 55° . Find the other angle and the lengths of the other two sides.
- (c) (*) Use Maple to find **all** of the solutions of $18 \sin x + x - 1 = 0$. Hint: plot a graph of the function first. [10]

Linear algebra

9. Matrices A and B are defined as follows $A = \begin{pmatrix} 1 & 5 & -4 \\ 4 & -1 & 3 \\ 0 & 0 & -2 \end{pmatrix}$ $B = \begin{pmatrix} 6 & 1 & 5 \\ -2 & 0 & 3 \\ 1 & 1 & -2 \end{pmatrix}$
- Find $A - B$, AB and BA . [8]
10. Find the inverse of each of the following matrices:

$$C = \begin{pmatrix} 3 & 8 \\ 0 & 1 \end{pmatrix} \quad D = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \quad E = \begin{pmatrix} 7 & 7 \\ 3 & -1 \end{pmatrix}. \quad [6]$$

11. Solve the following system of simultaneous equations using Gaussian elimination [8]

$$\begin{aligned}x_1 + x_2 &= 7 \\x_2 - x_3 + x_4 &= 5 \\x_1 - x_2 + x_3 + x_4 &= 6 \\x_2 + x_4 &= 10\end{aligned}$$

12. (*) Find the inverse of the matrix M, where

$$M = \begin{pmatrix} 2 & 5 & 9 \\ 8 & 11 & 7 \\ 4 & 6 & 13 \end{pmatrix}.$$

(Hint: use Maple) Hence, solve the following system of simultaneous equations: [8]

$$\begin{aligned}2x + 5y + 9z &= 4 \\8x + 11y + 7z &= 7 \\4x + 6y + 13z &= 1\end{aligned}$$