COM Mathematical Methods for Computing 2009

COURSEWORK II

To be handed in at the undergraduate office no later than 1500 on Friday December 18

You can and indeed should use Maple to assist your calculations wherever possible and to check your answers, but marks will **only** be given if **all** your relevant working is shown This coursework counts for 15% of the marks for this module.

Your solutions must be all your own work.

Equations

1. $f(x) = x^2 + 4x + 3$. Find the roots of $f(x) = 0$. Hence write down the roots of
(a) $f(x - a) = 0$, (b) $f(3x) = 0$ (c) $f(\frac{x}{2}) = 0$ (6)

2. Find all the solutions to the following equations:

   (a) $4\sin^3 x - 2\sin^2 x - 6\sin x + 3 = 0$ for $x \in [0, 2\pi]$
   Hint: factorise the left hand side. (3)

   (b) $x^6 + 9x^3 + 8 = 0$ Hint: look for a suitable substitution. (3)

   (c) $e^{2x} + 6e^{-x} - 7 = 0$ Hint: express the left hand side in positive powers of e^x, look for a suitable substitution and then factorise. (4)

Linear algebra

3. The vectors $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ are defined as follows:

$$\mathbf{a} = (3, 5, 7), \quad \mathbf{b} = (2, -1, -5), \quad \mathbf{c} = (-2, 3, 1).$$

   Evaluate the following:

   (a) (i) $\mathbf{a} - \mathbf{b}$ (ii) $\mathbf{b} + \mathbf{c} - \mathbf{a}$ (iii) $2\mathbf{c} + 3\mathbf{b}$ (3)

   (b) (i) $\| \mathbf{a} \|$ (ii) $\| 2\mathbf{c} + 3\mathbf{b} \|$ (3)

   (c) (i) $\mathbf{a} \cdot \mathbf{b}$ (ii) $(\mathbf{a} + \mathbf{b}) \cdot (\mathbf{c} - \mathbf{a})$ (4)

4. Find the eigenvalues and the corresponding eigenvectors of the matrix M and of its inverse, where

$$M = \begin{pmatrix} 2 & 3 \\ -1 & 6 \end{pmatrix}.$$

   Comment on your solutions. (10)

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Differentiation

5. Find the derivatives of each of the following functions in their simplest form:

   (a) (i) $y = 7x^4 + 3x$, (ii) $y = (1-3x)^3$, (iii) $y = \dfrac{4}{(1-2x)^2}$, (iv) $y = \sqrt{1+3x}$. (4)

   (b) (i) $y = e^{-2x}$, (ii) $y = \cos(2x+1)$, (iii) $y = (1+e^x)^2$, (3)

   (c) (i) $y = x^2\sqrt{1+x}$, (ii) $y = \dfrac{2-x}{x^2+1}$, (iii) $y = \ln\tan x$. (6)

6. Prove that if $y = e^{-x}(\cos 4x - \sin 4x)$, then y satisfies the equation (6)

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 17y = 0.$$

7. Find the x and y coordinates of the turning points of the following functions and classify them as maxima, minima or points of inflection: (10)

$$\text{(a)} \quad y = -3x^2 + 2x + 1, \qquad \text{(b)} \quad y = x^3 - 9x^2 + 26x - 24.$$

8. Find the equation of the tangent to the curve $y = e^{-2x}(1+x)$ at the point where $x = 0$. (5)

9. Showing all your working, using the Newton Raphson method, solve the following equations:

   (a) $x^3 + x - 1 = 0$, correct to three decimal places. (5)

   (b) $\cos x = x^2$, (How many roots are there?) correct to four decimal places.
   Hint: sketch the graph of each side of the equation on the same plot. (5)

Series

10. (a) Find the sum of all odd positive numbers less than 3000. (2)

   (b) Evaluate the following:
   (i) $\displaystyle\sum_{n=1}^{10} 3^n$ (ii) $\displaystyle\sum_{n=0}^{\infty} \frac{2}{3^n}$ (iii) $\displaystyle\sum_{n=4}^{11} 2^n$ (6)

   (c) Find the coefficient of x^5 in the binomial expansion of each of the following:

   (i) $(3x-2)^7$ (ii) $(4x-5)^6$ (4)

   (d) Find the first four terms in ascending powers of x and state the range of values of x for which the series is valid in each case:

   (i) $\dfrac{1}{1+2x}$ (ii) $\dfrac{1}{\sqrt{1-3x}}$ (iii) e^{-2x} (iv) $\sin(2x^2)$ (8)