MS112 Quantitative Methods

Exercise Sheet 5

1. Work through as many of the examples and exercises in the Differentiation Workbook as you can.

2. Find the derivative $\dfrac{dy}{dx}$ in each of the following cases, simplifying your answers:

   (a) $y = 3x^4$, (b) $y = \frac{1}{2}x^{\frac{8}{3}}$, (c) $y = 5x^4 + x^3 + \dfrac{2}{x}$, (d) $y = \cos 6x$, (e) $y = 2e^{-3x}$,

   (f) $y = \sqrt{x} + \ln x^2$, (g) $y = x^4 e^{2x}$, (h) $y = \dfrac{1 - \cos x}{x}$, (i) $y = \dfrac{\cos x}{\sin x}$,

   (j) $y = e^{-2x}(2x^2 + 2x - 1)$, (k) $y = \ln(1 + 2x^2)$, (l) $y = e^{\sin x}$.

3. For each of the following functions, find the co-ordinates of all the local maxima and minima:

   (a) $-3x^2 + 4x + 5$, (b) $y = 2x^3 - 5x^2 + 4x - 1$ (c) $y = x^4 - 2x^2$, (d) $y = x^2 + \dfrac{250}{x}$.

4. Find the coordinates of the points of inflection of the curve $y = x^4 - 6x^3 + 12x^2 + 5x - 3$.

5. Find the equations of the tangent and the normal to the curve $y = e^{3x}(1 - x)$ at the point where $x = 0$.

6. Find the second derivative $\dfrac{d^2y}{dx^2}$ of (a) $y = \ln(1 + x^2)$, (b) $y = e^x \cos x$

7. Find and characterise the turning points of $y = e^{-x^2}$. Sketch the graph.

More Challenging Questions:

8. The binomial expansion of $(x + h)^n$ is

$$(x + h)^n = x^n + nx^{n-1}h + \frac{n(n-1)}{2}x^{n-2}h^2 \ldots$$

   Using these first three terms show that when $y = x^n$, then $\frac{dy}{dx} = nx^{n-1}$, from first principles.

9. Find the first and second derivatives of $y = e^{-2x} \sin 5x$ and hence show that this function satisfies

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 29y = 0.$$

10. Find and characterise all the turning points of $y = x^3 e^{-x}$. Sketch the graph.