UNIVERSITY OF SURREY©

B. Sc. Undergraduate Programmes in Computing

Level HE2 Examination

Module MS214 Computational Operations Research

Time allowed – 2 hrs Spring Semester 2008

Attempt THREE questions.

If a candidate attempts more than THREE questions
only the best THREE questions will be taken into account.

Question 1

A factory produces two kinds of explosives for mining and quarrying, Type I and Type II.

Due to storage problems a maximum of 250 kilograms of explosive can be processed, mixed and packed each week. One kilogram of Type I explosive takes 5 hours processing time and 2 hours mixing and packing time, while Type II explosive takes 2 hours to process and 5 hours to mix and pack. The total processing capacity is 1100 hours per week, the total mixing and packing capacity is 900 hours per week. The profit on one kilogram of Type I explosive is £3 and on one kilogram of Type II explosive is £4.

The Company wants to know how much of each explosive to manufacture in order to maximise their profit per week.

(a) What are the decision variables for this problem? [2]

(b) What is the objective function? [2]

(c) State the constraints as inequalities. [5]

(d) Sketch the feasible region, indicating the coordinates of each of the corner points. On your diagram, show the contour associated with a weekly profit of £1100. [5]

(e) Using this contour or otherwise (but NOT by using the Simplex Method), determine the optimum solution to the problem, justifying your approach. [4]

(f) How much of each type of explosive should the Company manufacture each week? [2]

(g) The demand for Type II explosives has risen greatly because of the global mining boom and the Company intends to increase its price. By how much can the profit on Type II explosive be increased before the optimal solution in part (e) changes? [3]

(h) If the profit on Type II explosives exceeds this amount, what is your recommendation to the company and why? [2]

Question 2

(a) Put the following problem in standard form. Do NOT attempt to solve it.

Minimize

$$z = -3x_1 - 4x_2 + 5x_3$$

subject to the constraints

$$\begin{aligned} 4x_1 + 2x_2 - 3x_3 &\geq 3 \\ -2x_1 + 3x_2 + x_3 &\leq -1 \end{aligned}$$

and

$$x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \quad \text{unrestricted}$$

[7]

(b) Consider the following problem:

Maximize

$$z = 5x_1 + 3x_2$$

subject to the constraints

$$\begin{aligned} 4x_1 + 2x_2 &\leq 8 \\ x_1 + 5x_2 &\leq 6 \end{aligned}$$

and

$$x_1 \geq 0, \quad x_2 \geq 0$$

(i) Write this problem in standard form. [2]

(ii) Find an initial basic feasible solution, stating which are the basic variables and which are the non-basic variables. [2]

(iii) Construct the initial tableau for the Simplex Method. [3]

(iv) Use the tableau form of the Simplex Method to find the optimal solution of the problem. At each stage, state the entering and departing variables from the basis and explain your reasoning. [10]

(v) From the optimal tableau, determine the optimal value of z and state the values of all the variables at this solution. [1]

Question 3

(a) The optimal tableau of a Linear Programming problem which has two decision variables and two slack variables associated with two constraints is as follows:

Basic	z	x_1	x_2	x_3	x_4	Solution
x_2	0	4	1	0	3	5
x_3	0	5	0	1	2	8
z	1	3	0	0	4	12

An additional constraint is to be added to the problem, given by

$$x_1 - 3x_2 + 3x_3 \leq 4.$$

(i) From the optimal tableau, express the basic variables in terms of the non-basic variables. [2]

(ii) Express the extra constraint in standard form involving a new slack variable x_5. [1]

(iii) Rewrite the new constraint in terms of the non-basic variables of the original optimal solution. [2]

(iv) Form a new tableau consisting of the optimal tableau with the extra constraint added. [4]

(v) Determine the entering and departing variables for this new tableau and give your reasoning. [2]

(vi) Find the new optimal value of z, assuming that it will be obtained from this first iteration of the Simplex method. [You do NOT have to find the complete optimal tableau.] [2]

(b) The demand for ink at Central Printing Works is 20 litres per day. The print shop manager is able to buy ink for £3 per litre for orders of less than 55 litres and £2.50 per litre for orders of 55 litres or more. Every time an order is placed, a fixed cost of £12 is incurred. The daily cost of storing ink is £0.30 per litre.

Determine the optimal order quantity and the associated cost per day. [12]

Question 4

Three warehouses are supplied by three factories. The supply available from each factory, the demand at each warehouse and the cost per unit of transporting goods from the factories to the warehouses are summarised in the following table:

	Warehouse 1	Warehouse 2	Warehouse 3	Supply
Factory 1	3	6	4	12
Factory 2	5	2	6	16
Factory 3	4	3	3	14
Demand	10	8	24	

(i) What is meant by a *balanced* transportation model? [1]

(ii) Briefly describe how a transportation model with excess demand can be made into a balanced model, assuming that there is no penalty for unsatisfied demand. [3]

(iii) Is the above problem balanced or unbalanced? Briefly justify your answer. [2]

(iv) Use the North-West Corner Method to find an initial basic feasible solution of this problem. [Do NOT use the Least-Cost method.] [3]

(v) Find the optimal solution of this problem, i.e. the solution that minimises the transportation costs. [14]

(vi) What is the minimum total transportation cost? [2]

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