# Applying Spring-Relaxation Technique in Cellular Network Localization

Qing Zhang and Chuan Heng Foh
School of Computer Engineering
Nanyang Technological University
Singapore
Email: {Y060098, ASCHFoh}@ntu.edu.sg

Boon-Chong Seet
Department of Electrical
and Electronic Engineering
Auckland University of Technology
New Zealand

A. C. M. Fong
School of Computing and
Mathematical Sciences
Auckland University of Technology
New Zealand

*Abstract*—This paper presents a simple solution suitable for the mandatory localization function for E-911 services specified by FCC. Our solution introduces zero-length spring technique to compute the estimated location based on received signal strength (RSS). The introduced zero-length spring concept permits a less detailed path loss model to use without significant impact to the location estimation. We show the stability of our algorithm by illustrating the convergence of the estimated locations computed by the algorithm. We then demonstrate with simulation the accuracy of the estimation with various settings, and compare this accuracy with two candidate localization techniques.

*Index Terms*—Spring-relaxation technique localization, Location estimation, Received signal strength.

## I. INTRODUCTION

Localization in wireless networks has been an ongoing research with the goal of offering accurate localization functions to location-aware mobile applications [1]–[6]. In addition, the need for localization in cellular networks is promoted due to the FCC initiative of providing mandatory localization function for E-911 services for mobile subscribers in cellular networks [7].

Achieving localization in cellular networks often requires additional specialized hardware component in handsets and even base stations. A potential candidate of this approach is the use of GPS. While GPS solution requires additional hardware only in handsets, it is limited to outdoor environment. GPS solution in indoor environment may be enabled by having additional hardware components supported in base stations. However, the need for upgrading massively deployed infrastructure may appear as a hurdle to this solution.

Since all handsets have the ability to read received signal strength (RSS), using RSS for localization has become a potential approach for wireless networks including cellular networks [1]–[3], [6]. The main advantage of this approach is its ability to operate based on current hardware with no extra costs. However, to operate accurately, RSS-based localization requires prior information which may be the path loss model of the environment, or the wireless fingerprint obtained via RSS site survey.

In [8], Li proposes jointly estimation of location of a node and a path loss parameter of the environment concurrently. While this approach eliminates the need to obtain the key path loss parameter by other means, its use of nonlinear least square solution demands high computational power.

This paper presents an exploration of an alternative approach to the same problem specifically for cellular network. We use spring-relaxation technique [9] that is known to have lower complexity for localization. In addition, we propose zero-length springs as an extension of the technique. We find that path loss accuracy has less impact on the location estimation when zero-length springs are used in the spring-relaxation technique for localization. This property permits a less rigorous path loss parameter to be used without sacrificing much the accuracy of the estimated location. In the next section, some related work in the literature are reviewed. Section III presents our localization solution. Section IV provides analysis focusing on the convergence property of our solution. Section V presents the simulation results and compares individually the performance of our proposed solution with w-$k$NN and the spring-relaxation for localization, which are the two phases of the solution proposed previously in [10]. Finally, Section VI summarizes our conclusions.

## II. RELATED WORK

In general, sensor localization makes use of a limited number of beacons, whose locations are fixed and known, to estimate unknown locations of other nodes in a network. Supported by most of the existing transceiver chipsets, many localization systems proposed in literature make use of RSS through different techniques for location estimation. Some localization systems require prior information such as the path loss model of the working environment to estimate distances between transmitters and receivers based on RSS. Then, these distance estimates are used to compute unknown locations of nodes using different techniques, such as lateration and triangulation [3]. It is investigated in [3] that the ranging errors due to noisy RSS can be mitigated through refinement phases. However, these techniques are inevitably vulnerable to ranging errors due to unpredictable radio propagation behavior and difficulty in deducing accurate path loss model. Some localization systems require prior information such as the wireless fingerprint obtained via RSS site survey, examples include RADAR [1]. In RADAR, prior to the localization, RSS surveys are done at a number of points to construct the wire-

less fingerprint database. During the localization, given RSS information at an unknown location, the $k$-nearest neighbors ($k$NN) approach is used to search within the database for a set of closest matching locations, and to compute the location estimation for the unknown location based on the identified set. The main drawback of this approach is its need to perform survey, and adequate survey must be conducted to maintain a certain accuracy of the localization.

In our previous work [10], a hybrid RF mapping and ranging based localization solution was proposed for considered WSN. The proposed solution, consisting of two phases, harnesses the strengths of RF mapping and cooperative ranging, to overcome the potential weaknesses in one another. Phase 1 is an initial location estimation using a coarse-grained RF map based on weighted $k$-nearest neighbors (w-$k$NN), which is similar to the technique that RADAR [1] adopts. Given prior information of the wireless fingerprint obtained via RSS site survey, $k$ neighbors whose RF fingerprints are the closest matching to that of a sensor in signal space, are identified among all pre-mapped survey points. Then, each of these $k$ neighbors is weighed inversely proportional to their Euclidean distance in signal space from the sensor in the estimation of its location. Phase 2 is a cooperative ranging-based refinement using spring-relaxation technique. In [9], a spring-relaxation algorithm was designed for large-scale Internet to predict the communication latency measured by round trip time between the hosts in the network. Inspired by [9], we extend and introduce a spring-relaxation technique to localization problem. In phase 2, given prior information of the path loss model of the environment, each sensor ranges to each of its neighbors, and uses the distance estimated from RSS to iteratively refine its own location. It is shown in [10] that the proposed solution is not only suitable for localization problem in WSN, but also able to provide satisfactory accuracy, if detailed path loss information is available. However, if only less detailed path loss information is available, the spring-relaxation technique will suffer significant performance degradation because of erroneous ranging.

To overcome this problem, in this paper we propose a zero-length spring technique especially for cellular network localization. The zero-length spring technique is an extension of the spring-relaxation technique proposed in [10] in a way that all springs will have zero natural length. By setting natural length to zero, all springs will only deal with extension force, but no compression force, so that the negative impact of less detailed path loss model can be mitigated. Additionally, springs are designed to have spring constants proportional to corresponding RSS, so that a spring connecting a nearer neighbor to a particular mobile node will apply a stronger force on that node.

## III. ALGORITHM

We shall first explain the concept of spring-relaxation technique for localization, then describe our system setup followed by our proposed solution in details.

The phase 2 of our localization solution proposed in [10] transforms the problem of estimating unknown locations into finding the stopping points of moving particles in a spring network. Specifically, in a spring network consisting of fixed pins, moving particles, and springs, each spring attaches a particular particle to one of its one-hop neighbors (either pins or other particles) in a stretched or compressed way. With effect of forces produced in springs, each particle fluctuates in the spring network will eventually stop at a stopping point where the net force applied on that particle is zero. In the considered WSN in [10], the proposed localization solution considers beacons as pins in fixed and known locations, and sensor nodes as particles in initial guessed locations. Virtual springs attach nodes to their one-hop neighbors (either beacons or other nodes), whose natural length are the estimated distances in between according to RSS measurements. To simulate the process of particles moving towards their stopping locations, the forces in springs, movements of nodes with the effect of forces, and current locations of nodes are computed and updated continuously in iterations using a set of linear equations. Only until the net force exerted on a particular node released to zero, that node will stop at a point which is its estimated location.

In this paper, we propose zero-length springs as an extension of the spring-relaxation technique in [10] for cellular network localization. We find that path loss accuracy has less impact on the location estimation when zero-length springs are used in the spring-relaxation technique.

Our considered cellular network consists of a number of mobile nodes randomly placed on a map of predefined size with a number of base stations. As shown in Fig. 1, every mobile node communicates to all of its visible base stations and neighboring nodes to measure the RSS and collect their location information. After collecting RSS and locations of all its visible base stations and neighboring nodes, the mobile node executes the algorithm to estimate its location based on the proposed zero-length spring technique.

Let $\mathbb{MN}$ and $\mathbb{BS}$ be the sets describing all mobile nodes and base stations respectively. Each mobile node is noted as $Node_i$, with its location noted as $\overrightarrow{V_i}$, $i \in \mathbb{MN}$. Each base station is noted as $Node_j$, with its location noted as $\overrightarrow{V_j}$, $j \in \mathbb{BS}$. For simplicity in presentation, we use $Node_p$, $p \in (\mathbb{MN} \cup \mathbb{BS})$ to represent all the mobile nodes and base stations as a whole in the cellular network. Let $k_{i,p}$ be the spring constant of the spring connecting $Node_i$ and $Node_p$, which is proportional to the signal strength $RSS_{i,p}$ that $Node_i$ received from $Node_p$. Define $\overrightarrow{F_{i,p}}$ to be the force that the spring between $Node_i$ and $Node_p$ exerts on $Node_i$. Since the natural length of each spring is zero, based on Hooke's law, we have

$$\overrightarrow{F_{i,p}} = k_{i,p}(\overrightarrow{V_p} - \overrightarrow{V_i}).$$

The net force on $Node_i$, defined as $\overrightarrow{F_i}$, is the vector sum of all forces

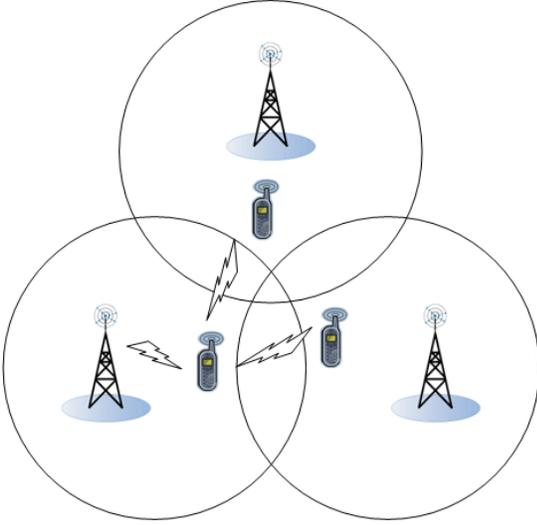$$\overrightarrow{F_i} = \sum_{p \in (\mathbb{MN} \cup \mathbb{BS})} \overrightarrow{F_{i,p}}.$$

Fig. 1. A mobile node communicates to all of its visible base stations and neighboring nodes to measure the RSS and collect their location information.

To mimic the evolution of the spring network, our algorithm updates the locations of mobile nodes in iterations. In each iteration, the algorithm moves $Node_i$ a small distance in the direction of $\overrightarrow{F_i}$ and then recomputes all the applied forces. Let $\delta$ be the step size of location adjustment. Considering a linear relationship between the net force and the displacement, the location of $Node_i$ is then updated as

$$\overrightarrow{V_i} \leftarrow \overrightarrow{V_i} + (\delta \cdot \overrightarrow{F_i}).$$

Algorithm 1 describes the procedure for $Node_i$ to estimate its location. It is expected that with the contribution of location information from its neighboring nodes besides that from its visible base stations, a mobile node can get a more accurate location estimate.

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**Algorithm 1** Zero-Length Spring Technique

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  INPUT: $RSS_{i,p}$ and $\overrightarrow{V_p}$
  OUTPUT: location estimates of $\overrightarrow{V_i}$
  **while** $\|\overrightarrow{F_i}\| \geq \tau$ **do**
    $\overrightarrow{F_i} = 0$
    **for all** $Node_p$ **do**
      **if** $Node_p$ is visible to $Node_i$ **then**
        $\overrightarrow{F_{i,p}} = k_{i,p}(\overrightarrow{V_p} - \overrightarrow{V_i})$
        $\overrightarrow{F_i} = \overrightarrow{F_i} + \overrightarrow{F_{i,p}}$
      **end if**
    **end for**
    $\overrightarrow{V_i} = \overrightarrow{V_i} + \delta \cdot \overrightarrow{F_i}$
  **end while**

---

There are several design parameters which are used to control the execution of Algorithm 1. Threshold is a constant that used to define the visibility of a node. If the received signal strength $RSS_{i,p}$ from $Node_p$ to $Node_i$ is no smaller than the threshold, then $Node_p$ is visible to $Node_i$. Tolerance

$\tau$ is involved in the termination condition for the **while** loop. The **while** loop does not stop until the magnitude of the net force $\|\overrightarrow{F_i}\|$ is smaller than $\tau$. Step size $\delta$ controls the proportion of the net force actually transferred onto a mobile node in one iteration. In other words, $\delta$ controls the convergence speed of the algorithm.

## IV. SYSTEM MODEL

Consider a system consisting of $n$ mobile nodes. For simplicity, we assume that a mobile node, called $Node_i$, can detect the RSS of all its $n-1$ neighbors during the execution of the algorithm. Note that this is not a necessary condition for the algorithm to operate, and this is automatically achieved if the movements of the nodes are slow such that the algorithm ends before any neighbor of $Node_i$ losses contact.

Let $\mathbb{J}_i$ denote the set of neighbors of $Node_i$. Let $\overrightarrow{F_i(t)}$ be the net force applied on $Node_i$ from all nodes in $\mathbb{J}_i$ at time $t$, and $n_j = |\mathbb{J}_i|$. The net force $\overrightarrow{F_i(t)}$ can be expressed as

$$\overrightarrow{F_i(t)} = \sum_{j \in \mathbb{J}_i} \overrightarrow{F_{i,j}(t)} \tag{1}$$

where $\overrightarrow{F_{i,j}(t)}$ is the force applied on $Node_i$ from $Node_j$. Let $F_i^{(x)}(t)$ and $F_i^{(y)}(t)$ be the x- and y-components of $\overrightarrow{F_i(t)}$ respectively. Likewise, let $F_{i,j}^{(x)}(t)$ and $F_{i,j}^{(y)}(t)$ be the x- and y-components of $\overrightarrow{F_{i,j}(t)}$ respectively. Considering zero-length spring, based on Hooke's law, we have

$$\begin{cases} F_{i,j}^{(x)}(t) = k_{i,j}(x_j(t) - x_i(t)) \\ F_{i,j}^{(y)}(t) = k_{i,j}(y_j(t) - y_i(t)) \end{cases} \tag{2}$$

where $(x_i(t), y_i(t))$ and $(x_j(t), y_j(t))$ are the coordinations of $Node_i$ and $Node_j$ respectively at time $t$, and $k_{i,j}$ is the spring constant, which is proportional to the RSS $Node_i$ received from $Node_j$. Without loss of generality, we shall now focus on the x-component of evolution of the forces and location. According to our algorithm, in each iteration, the estimated location is adjusted based on the previous location plus a certain degree, $\delta$, of the applied net force, which gives

$$x_i(t + \Delta t) = x_i(t) + \delta F_i^{(x)}(t)\Delta t.$$

Rearranging above yields

$$\frac{x_i(t + \Delta t) - x_i(t)}{\Delta t} = \delta F_i^{(x)}(t).$$

Consider continuous time evolution of our system, applying $\Delta t \to 0$ limit to both sides of the above results, we have

$$x_i'(t) = \delta F_i^{(x)}(t). \tag{3}$$

Combining (1)-(3), we get

$$x_i'(t) = \delta \sum_{j \in \mathbb{J}_i} (k_{i,j}(x_j(t) - x_i(t))) \tag{4}$$

where the motion of the location can be described by a linear dynamical system. In the following subsections, we shall use this result to show the convergence of our algorithm. In other words, we shall show that the magnitude of the changes of

the consecutive estimated locations decreases monotonically over the iterations and this monotonic decrease ensure that the estimated location converges to a particular coordinate over the iterations of the algorithm.

With our algorithm operates in a cooperative manner, for its location estimation, $Node_i$ may use information collected from nodes whose locations are also being estimated. Such a system gives rise to a set of simultaneous equations of (4). Consider a single hop environment where each of the $n$ mobile nodes is visible to all others, by (4), we can describe the system by

$$
\begin{cases}
x_1'(t) & = \delta\left(-\widehat{k}_1 x_1(t) + k_{1,2}x_2(t) + \cdots + k_{1,n}x_n(t)\right) \\
x_2'(t) & = \delta\left(k_{2,1}x_1(t) - \widehat{k}_2 x_2(t) + \cdots + k_{2,n}x_n(t)\right) \\
\quad \vdots \\
x_n'(t) & = \delta\left(k_{n,1}x_1(t) + k_{n,2}x_2(t) + \cdots - \widehat{k}_n x_n(t)\right)
\end{cases}.
$$

Recall that $x_i(t)$ is the x-coordination of $Node_i$ at time $t$, and $k_{i,j}$ is the spring constant proportional to $RSS_{i,j}$ that $Node_i$ received from $Node_j$. $\widehat{k}_i = \sum_{j \in \mathbb{J}_i} k_{i,j}$ is also a constant. We can represent these simultaneous differential equations in matrix form as

$$
\frac{d\mathbf{X}}{dt} = \mathbf{M}\mathbf{X} \tag{5}
$$

where

$$
\mathbf{X} = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix}
$$

and

$$
\mathbf{M} = \delta \begin{bmatrix}
-\widehat{k}_1 & k_{1,2} & \ldots & k_{1,n} \\
k_{2,1} & -\widehat{k}_2 & \ldots & k_{2,n} \\
\vdots & \vdots & \vdots & \vdots \\
k_{n,1} & k_{n,2} & \ldots & -\widehat{k}_n
\end{bmatrix}. \tag{6}
$$

From (6), it can be seen that $\mathbf{M}$ is a singular matrix since $|det(\mathbf{M})| = 0$. According to the design, all entries of $\mathbf{M}$ are positive except for those along the diagonal, for the spring constants are designed to be proportional to RSS. Suppose the RSS between a mobile node and its neighbor is bidirectional, so that $k_{i,j} = k_{j,i}$ which suggests $\mathbf{M}$ is also symmetric. Based on these characteristics, the solution of (5) takes the following form

$$
\mathbf{X} = \sum_{i=1}^{n} c_i V_i e^{D_i t}
$$

where $c_i$ are arbitrary constants, $D_i$ are eigenvalues of the matrix $\mathbf{M}$ which are real numbers, and $V_i$ are corresponding eigenvectors of $\mathbf{M}$.

Consider the quadratic form of $\mathbf{M}$ given by

$$
Q(w) = w^T \mathbf{M} w
$$

| Variable | Definition |
|---|---|
| $P_{TX}$ | transmitted power level [dBm] |
| $P_{RX}$ | received power level [dBm] |
| $G_{TX}$ | antenna gain of the transmitter [dBi] |
| $G_{RX}$ | antenna gain of the receiver [dBi] |
| $n$ | path loss exponent |
| $d$ | distance between the transmitter and the receiver [m] |
| $\lambda$ | signal wavelength [m] |
| $X_\alpha$ | Gaussian random variable with a standard deviation of $\alpha$ |

where $w \in \mathbb{R}^n$ is an arbitrary nonzero vector. Expanding and arranging $Q(w)$ yields

$$
Q(w) = \sum_{i=1}^{n} \sum_{j=i+1}^{n} -k_{i,j}(w_i - w_j)^2
$$

and the result proves that $\mathbf{M}$ is a negative definite matrix whose eigenvalues are all negative. The evolutions of the locations on the y-axis for all nodes have the same expressions as that on the x-axis. The exponential decay of the estimated location indicated in the results shows the convergence of the estimated location in a cooperative operational manner.

## V. PERFORMANCE

In this section, simulation experiments are conducted to study the accuracy of the proposed solution in Matlab. Further experiments are carried out to show that the solution is especially favorable under the circumstances where precise path loss model is not available by comparing individually with the two phases of the solution proposed in [10] in the same simulation environment.

The environment for simulation experiments is a coordinated map of size 1500 m × 1500 m. Five base stations are placed in the four corners and the central point of the map, with a radio coverage of 1000 m. 150 mobile nodes are randomly deployed into the map so that each node will have 10 neighboring nodes in average. We use the following path loss model for the radio propagation [11].

$$
\begin{aligned}
P_{RX} = \, & P_{TX} + G_{TX} + G_{RX} - 10n \log_{10} d + \\
& + 20 \log_{10} \lambda - 20 \log_{10}(4\pi) - X_\alpha
\end{aligned} \tag{7}
$$

where the involved variables are described in Table I, and their default values follow the GSM-900 specification for cellular network [11].

### A. Performance with Correct Prediction of Path Loss Exponent

The performance of the zero-length algorithm is evaluated in the environment with a Gaussian noise $N(0, 5^2)$, and a path loss exponent of 3 for outdoor environment (i.e. $\alpha = 5$ and $n = 3$ in Table I). Assume detailed path loss information is available, then this path loss exponent can be predicted correctly at 3. A location estimation error for $Node_i$, $i \in \mathbb{MN}$
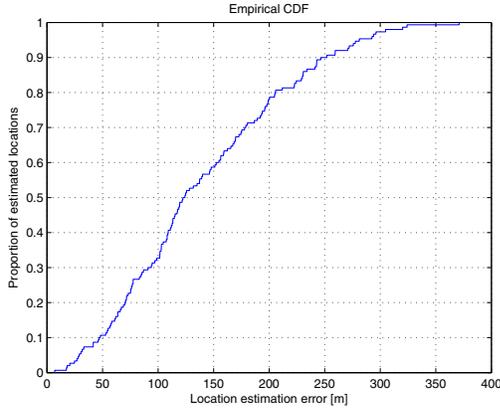
Fig. 2. Empirical CDF of the zero-length spring technique.

| Number of survey points | Mean location estimation error [m] |
|---|---|
| 100 | 268.9798057 |
| 2,500 | 284.0504617 |
| 10,000 | 280.7679469 |
| 40,000 | 262.3167403 |
| 90,000 | 282.4347595 |
| 160,000 | 290.7683059 |
| 250,000 | 252.4084913 |

| Actual path loss exponent | Predicted path loss exponent | Zero-length algorithm | Spring-relaxtion technique |
|---|---|---|---|
| 3 | 2 | 147.3280936 | 9548.340925 |
| 3 | 4 | 141.8453962 | 234.816221 |
| 3 | 5 | 143.3554491 | 250.8288482 |
| 3 | 6 | 151.1707408 | 242.1384056 |

is measured by the distance between its estimated location and true location with the following equation.

$$error = \sqrt{(x_{e,i} - x_i)^2 + (y_{e,i} - y_i)^2}$$

where $(x_{e,i}, y_{e,i})$ is the estimated location and $(x_i, y_i)$ is the true location of $Node_i$ in the map.

Simulation experiments are carried out to examine the mean location estimation error over all 150 mobile nodes in the map. Numerically, the zero-length spring algorithm produces a mean error of 140.4972 m, which is about 10% of the map length. To get a better idea of the performance, Fig. 2 plots the empirical CDF of the location estimation errors of all nodes in the map.

We further compare the zero-length algorithm with the w-$k$NN approach which is phase 1 of the proposed solution in [10] with the same setup in the same environment. We set $k = 3$ for the w-$k$NN in the simulation, so that the estimated location of a particular mobile node will be computed based on the nearest 3 neighboring survey points. We gradually increase the number of survey points from 100 to 250,000, where these survey points are deployed in cells of equal width and height in the map. The performance of the w-$k$NN approach is summarized in Table II by examining the mean location estimation error. From Table II, we observe no significant performance improvement of the w-$k$NN approach with the rapid increase of the number of survey points. Additionally, the best case of the mean location estimation error among these results is still 100 m larger than that of the zero-length algorithm. This observation indicates that the zero-length solution can provide an estimation result better than the w-$k$NN approach in normal setup, without the requirement of time consuming surveys.

*B. Performance with Incorrect Prediction of Path Loss Exponent*

In many real applications, only less detailed path loss model is available, which introduces a problem of incorrect prediction of path loss exponent. According to the detailed description of the zero-length algorithm in Section III, it can

be concluded that the solution will provide similar results in the presence of incorrect prediction of path loss exponent, for the erroneous ranging has little impact on the solution. On the contrary, ranging-based techniques such as spring-relaxation approach [10] will suffer significant performance degradation under this circumstance. For comparison purpose, experiments of the spring-relaxation technique are conducted together with the cooperative case of the zero-length algorithm in the same environment with the presence of incorrect prediction of path loss exponent, and respective mean location estimation errors are summarized in Table III. It can be observed that with the incorrect prediction of path loss exponent, the performance of the spring-relaxation technique is much more sensitive than zero-length spring technique. As a result, the zero-length algorithm is favorable under this circumstance where detailed path loss information is not available.

## VI. CONCLUSION

In this paper, we proposed a zero-length spring localization solution which is an extension of spring-relaxation technique for cellular network. Our solution introduces zero-length spring technique to compute the estimated location based on received signal strength. The introduced zero-length spring concept permits a less detailed path loss model to use without significant impact to the location estimation. We showed that zero-length technique is suitable for cellular network by analyzing the convergence property of the algorithm. In our simulation experiments, we demonstrated the overall accuracy of our design and compared that with other relevant techniques in the same environment. Our simulation results have shown encouraging performance benefit of our solution, especially in

the scenario where only less detailed path loss information is available.

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