

# Collision Codes: Decoding Superimposed BPSK Modulated Wireless Transmissions

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**Abstract**—The introduction of physical layer network coding gives rise to the concept of turning a collision of transmissions on a wireless channel useful. In the idea of physical layer network coding, two synchronized simultaneous packet transmissions are carefully encoded such that the superimposed transmission can be decoded to produce a packet which is identical to the bitwise binary sum of the two transmitted packets. This paper explores the decoding of superimposed transmission resulted by multiple synchronized simultaneous transmissions. We devise a coding scheme that achieves the identification of individual transmission from the synchronized superimposed transmission. A mathematical proof for the existence of such a coding scheme is given.

## I. INTRODUCTION

The concept of physical-layer network coding (PNC) first introduced in 2006 by Zhang *et al.* reveals the idea of decoding a transmission collision on a wireless channel [1]. This concept directly challenges the *traditional rule* that a collided transmission on a wireless channel is undecodable. In this pioneering work, it has been demonstrated that a collision of two simultaneous wireless transmissions can be turned into a useful transmission. In brief, two simultaneous wireless transmissions that are added together at the electromagnetic wave level can be decoded and mapped to produce an outcome such that the relationship between the transmitted and the decoded binary information follows the exclusive-or (XOR) principle.

The concept of PNC is found to have potential in enhancing performance of the current wireless networks. In the original paper describing PNC in [1], the concept is demonstrated to be effective for further performance enhancement of network coding operation in wireless environment [6], [7]. In [2] the suitability of PNC is shown to improve the throughput capacity of a random wireless network by a fixed factor. Modulation and mapping schemes are proposed to allow for the decoding of a collision from two simultaneous transmissions. However, the scheme is only suitable for decoding a superimposed transmission consisting only two simultaneous transmissions within, and this greatly limits its applications.

Recently, Durvy *et al.* [3] applies the idea of decoding superimposed signals in PNC to improve the reliability of wireless broadcasting. While traditional approaches often attempt to avoid collisions [4], [5], in [3], the authors propose modified ARQ operation to invite collision of multiple acknowledgement (ACK) transmission and then design a scheme

to decode the collided ACK transmissions. Their idea is that upon receiving a broadcast transmission, each receiver detecting the transmission replies with an ACK transmission. These simultaneous ACK transmissions will cause a collision. Using the concept of PNC, decoding of the superimposed ACK transmissions is performed to identify the ACK transmitters. Their method assumes synchronous simultaneous ACK transmissions and the capability of precise detection of signal energy. Since simultaneous ACK transmissions appear after the completion of a broadcast transmission which is a common event, simultaneous ACK transmissions may be considered synchronized to a certain extent. However, the requirement of precise detection of signal energy for decoding introduces difficulty in the practical design.

We here define *collision decoding* to be the capability of the identification of individual transmission from the superimposed transmissions. In other words, it is the capability for the collision decoder to tell which set of stations has transmitted and which has no transmitted in a superimposed transmission. A particular application of this capability has been shown in [3] to be beneficial in protocol design and enhancement. However in [3], the need for precise detection of signal energy for decoding has reduced the practicability of the proposal. In this paper, we devise a coding scheme for collision decoding, that particularly addresses the challenge of signal energy detection. Our scheme overcomes the shortcoming of the method given in [3]. We also demonstrate with a mathematical proof the existence of such a coding scheme.

The rest of the paper is organized as follows. Section II revisits the PNC technique and describes our proposed scheme. In Section III, we give the mathematical proof that our proposed coding scheme allows unique identification of different receiver combinations. Finally, important conclusion is drawn in Section IV.

## II. DECODING A COLLISION

### A. Physical-Layer Network Coding

The concept of PNC was first introduced by Zhang *et al.* in [1]. The authors proposed a frame-based Decode-and-Forward (and amplify if necessary) strategy in packet forwarding. In their scenario, two neighboring nodes transmit simultaneously to a common receiver. Assuming perfect transmission synchronization in physical layer, based on the additive nature of simultaneously arriving electromagnetic

TABLE I  
THE PNC MAPPING FOR TWO TRANSMITTING NODES USING BPSK.

Modulation mapping at $N_1$ and $N_3$				Demodulation mapping at $N_2$	
Input		Output		Input	Output
$s_1$	$s_3$	$a_1$	$a_3$	$a_1 + a_3$	$s_2$
1	1	1	1	2	0
0	1	-1	1	0	1
1	0	1	-1	0	1
0	0	-1	-1	-2	0

(EM) waves, the receiver detects the added signal of the two transmitted modulated signals. Using a suitable mapping scheme, they show that for certain modulation schemes, there exists a mapping scheme such that the relationship between the two transmitted binary bits and the decoded binary bit follows the XOR principle.

We here revisit the PNC operation [1] and its mapping scheme to achieve the XOR principle. Consider two senders,  $N_1$  and  $N_3$ , and a common receiver  $N_2$ . Let  $s_1$  and  $s_3$  be the binary bit transmitted by  $N_1$  and  $N_3$  at a particular time respectively, and  $s_2$  be the decoded binary bit. Based on BPSK modulation, we have

$$r_2(t) = a_1 \cos(\omega t) + a_3 \cos(\omega t) = (a_1 + a_3) \cos(\omega t) \quad (1)$$

where  $r_2(t)$  is the received signal,  $a_1$  and  $a_3$  are the transmitted amplitudes, and  $\omega$  is the carrier frequency. For BPSK, we have  $a_j = 2s_j - 1$  [1]. At  $N_2$ , a scheme (see Table I) that maps a strong energy signal to binary 0 (i.e.  $|a_1 + a_3| = 2$ ) and a weak energy signal (i.e.  $a_1 + a_3 = 0$ ) to binary 1 can be applied which gives

$$s_2 = s_1 \oplus s_3. \quad (2)$$

### B. Collision of Multiple BPSK Modulated Transmissions

Considering a collision consisting of an arbitrary number of simultaneous transmissions, according to Table I, retaining the XOR principle for the decoding requires precise detection of signal energy for the PNC mapping, which reduces its robustness. In [3], the authors proposed a scheme to deal with decoding of multiple simultaneous transmissions. The scheme proposes bit sequences of  $N+1$  bits that is suitable for decoding a collision of up to  $N$  simultaneous transmissions. Likewise, the main limitation of the proposed scheme is that it also requires precision of power level differentiation in the decoding procedure. Precisely, comparisons of analog received signals are needed for the operation, and the authors propose use of a delay line to store analog signal for the comparison purpose. If we relax the needs for power level differentiation, based on the additive nature of EM waves, considering the BPSK modulation scheme, it is not difficult to see that the BPSK demodulation process follows the majority principle.

To illustrate this, let us consider a simple case of three transmitters,  $N_i$  ( $i \in \{1, 2, 3\}$ ), and one common receiver,  $N_0$ . Let  $a_i$  be the amplitude of the BPSK signals corresponding to the transmitted binary information, and  $a_0$  be the detected amplitude of the BPSK signals. Based on the additive nature

TABLE II  
AN EXAMPLE OF BPSK DEMODULATION FOR THREE TRANSMITTERS.

Input	$N_1$	0	0	0	0	1	1	1	1
	$N_2$	0	0	1	1	0	0	1	1
	$N_3$	0	1	0	1	0	1	0	1
Output	$N_0$	0	0	0	1	0	1	1	1

of EM waves, we have  $a_0 = a_1 + a_2 + a_3$ . Considering a common design of a BPSK demodulator with a matched filter and a detection device, the demodulator produces 1 if  $a_0 > 0$  and 0 otherwise. Table II exhausts all possible inputs and shows the relationship between the input and the output binary information. As can be seen, the relationship follows the majority principle.

A closer examination of BPSK shows that any pair of inputs that holds different binary information is offset when added at the EM level. As a result, the remaining input decides the binary outcome. In the case of a tie, since the detected energy fails to reach a threshold after the matched filter, a consistent conclusion will be made. Without loss of generality, we assume it to be binary 0.

### C. Our Proposed Scheme for Collision Decoding

Using the majority principle, we design a coding scheme that enables the common receiver of a collided transmission to tell the presence of individual transmission involved in the collision. We first illustrate the application of collision decoding using multicast application, followed by the proposed coding scheme.

Consider a wireless network consisting of a basestation and a collection of stations within wireless coverage of the basestation. For the multicast transmission, the basestation broadcasts the transmission on the wireless channel and only the intended receivers will process the transmission. In our design, the basestation also embeds information in its data transmission to tell each individual intended receiver of its unique identifier.

It is possible that not all intended receivers detect the transmission due to, for example noise. If an intended receiver detects the broadcast transmission, it first extracts its identifier embedded by the basestation, say  $i$ , and immediately replies an ACK with a predefined unique bitstream,  $s_i$ , as part of ACK.

With the immediate replies of ACK transmissions from multiple receivers, a collision of ACK transmissions occurs. The basestation decodes the superimposed transmission using a BPSK demodulator and as discussed in the previous subsection, the decoded bitstream, say  $\mathbf{v}$ , will follow majority principle. We shall show that there exists a coding scheme such that the basestation produces a unique bitstream for a particular combination of the receivers' bitstreams. This unique bitstream enables the basestation to identify whether a particular intended receiver has replied ACK indicating the success delivery of the multicast packet to that receiver.

Consider the number of receiver,  $N$ , to be an odd number, and a bitstream has  $V$  bits. Let  $R = \frac{N+1}{2}$ . We first construct

TABLE III  
THE DECODED BITSTREAMS (BASED ON THE MAJORITY PRINCIPLE) OF  
DIFFERENT RECEIVER COMBINATIONS FOR  $N = 3$ .

Receiver Combination	Decoded Bitstream
$N_1$	110
$N_2$	101
$N_3$	011
$(N_1, N_2)$	100
$(N_1, N_3)$	010
$(N_2, N_3)$	001
$(N_1, N_2, N_3)$	111

a binary matrix of size  $N \times V$  such that each column contains exactly  $R$  of binary 1 and  $R - 1$  of binary 0, with a unique permutation. Exhausting all permutations given  $R$  number of binary 1 and  $R - 1$  number of binary 0 produces  $\binom{N}{R}$  unique patterns, and hence  $V = \binom{N}{R}$ . With this construction, the binary matrix holds  $N$  number of unique  $V$ -bit bitstreams, each of which will be assigned to a receiver. While our scheme uses bit sequences of  $\binom{N}{R}$  bits, this method remains adequate to support up to 15 stations as the required number of bits is below 8000 bits (or 1000 bytes), which is suitable for existing protocols to carry.

We give an example of  $N = 3$  in the following. According to our scheme, the binary matrix for  $N = 3$ ,  $\mathbf{M}_3$ , can be constructed as

$$\mathbf{M}_3 = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}.$$

Each row in the binary matrix represents the unique bitstream for each receiver to transmit in its ACK. In Table III, we show the assignment of bitstreams to the three receivers,  $N_1, N_2, N_3$ . Moreover, we provide the decoded bitstreams at the base station given all possible combinations of ACK replies. In this example, we see that a particular combination of receivers can be uniquely identified by the decoded bitstream.

### III. THE PROOF

In this section, we show that our proposed coding scheme guarantees the uniqueness of the decoded bitstream for a particular receiver combination for any odd value of  $N$ . In case that the number of receivers  $N$  is an even number, we can use  $N + 1$  codes for the same purpose, and the additional bitstream in the codes will not be used.

Consider  $N$  stations, based on our design, we can construct a coefficient matrix  $\mathbf{M}_N$ , where each of its elements is either  $+1$  or  $-1$ <sup>1</sup>. Let  $G = \{1, \dots, N\}$  be the set containing all stations and  $R = \frac{N+1}{2}$ . According to our design, each of the column in  $\mathbf{M}_N$  contains exactly  $R$  number of  $+1$  and  $R - 1$  number of  $-1$ . In other words, for a given column  $c$ ,  $\sum_{r \in G} \mathbf{M}_N(r, c) = 1$ . Based on this design, exhaustive permutation of  $+1$  and  $-1$  for a column construction gives  $\binom{N}{R}$  unique patterns. The coefficient matrix  $\mathbf{M}_N$  that holds non-repetitive patterns of columns thus has a size of  $N \times \binom{N}{R}$ .

<sup>1</sup>Here we use  $+1$  and  $-1$  to replace 1 and 0 for better illustration.

We define  $F(G_0, c) = \sum_{r \in G_0} \mathbf{M}_N(r, c)$  where  $G_0 \subseteq G$ . In other words, the function  $F(G_0, c)$  gives the sum of the values corresponding to a given column  $c$  and a particular collection of stations  $G_0$ . Let  $\Omega_+(G_0, c)$  (resp.  $\Omega_-(G_0, c)$ ) denote the function that counts the number of  $+1$  (resp.  $-1$ ) corresponding to the the column  $c$  and the collection of stations given in  $G_0$ . By definition, we have

$$F(G_0, c) = \Omega_+(G_0, c) - \Omega_-(G_0, c). \quad (3)$$

We further use the notation  $|G_0|$  to denote the cardinality of the set  $G_0$ ,  $G'_0 = G \setminus G_0$  to denote the complementary set of  $G_0$ , and  $\vec{G}_0$  to denote a vector whose elements are given by

$$\vec{G}_0(c) = \begin{cases} 1, & F(G_0, c) \geq 1 \\ 0, & F(G_0, c) < 1 \end{cases} \quad (4)$$

where  $c = 1, 2, \dots, \binom{N}{R}$  and  $R = \frac{N+1}{2}$ .

Based on the above definition, we first have the following properties about  $F(\cdot)$ .

*Claim 1:* Let  $G_1, G_2 \subseteq G$  and  $G_1 \cap G_2 = \emptyset$ . If  $H = G_1 \cup G_2$ , then for all  $c$ ,  $F(H, c) = F(G_1, c) + F(G_2, c)$ .

*Claim 2:* Let  $G_1 \subset G_2 \subseteq G$ . If  $H = G_2 \setminus G_1$ , then for all  $c$ ,  $F(H, c) = F(G_2, c) - F(G_1, c)$ .

The above claims can be easily established by set operations. By definition,  $F(H, c) = \sum_{r \in H} \mathbf{M}_N(r, c)$ . Given that  $H = G_1 \cup G_2$ , we obtain

$$\begin{aligned} F(H, c) &= \sum_{r \in (G_1 \cup G_2)} \mathbf{M}_N(r, c) \\ &= \sum_{r \in G_1} \mathbf{M}_N(r, c) + \sum_{r \in G_2} \mathbf{M}_N(r, c) \end{aligned}$$

since  $G_1 \cap G_2 = \emptyset$ . With the above result, we have established claim 1. Claim 2 can be established with the same approach.

Since in our design, each column in  $\mathbf{M}_N$  contains exactly  $\frac{N+1}{2}$  number of  $+1$  and  $\frac{N-1}{2}$  number of  $-1$ , then by definition

$$\Omega_+(G, c) = \frac{N+1}{2} \Omega_-(G, c) = \frac{N-1}{2} F(G, c) = 1. \quad (5)$$

The above gives the following lemmas.

*Lemma 1:* Let  $G_1 \subset G$  where  $|G_1| = g$  and  $g$  is an odd integer  $\leq N - 1$ . There exists a column  $c$  in  $\mathbf{M}_N$  such that  $F(G_1, c) = 1$ .

*Proof:* By definition,  $\mathbf{M}_N$  holds exhaustive patterns of columns with exactly  $R$  number of  $+1$  and  $R - 1$  number of  $-1$ . In other words, any column with exactly  $R$  number of  $+1$  and  $R - 1$  number of  $-1$  is a column of  $\mathbf{M}_N$ .

Let  $G_1 \subset G$  where  $|G_1| = g$  and  $g$  is an odd integer  $\leq N - 1$ . We create a column  $c$  such that  $\Omega_+(G_1, c) = \frac{g+1}{2}$ ,  $\Omega_-(G_1, c) = \frac{g-1}{2}$  and  $\Omega_+(G'_1, c) = \Omega_-(G'_1, c) = \frac{N-g}{2}$ . Since  $G = G_1 \cup G'_1$  and  $G_1 \cap G'_1 = \emptyset$ , we yield

$$\begin{aligned} \Omega_+(G, c) &= \Omega_+(G_1, c) + \Omega_+(G'_1, c) = \frac{N+1}{2} = R \\ \Omega_-(G, c) &= \Omega_-(G_1, c) + \Omega_-(G'_1, c) = \frac{N-1}{2} = R - 1 \end{aligned}$$

which shows that  $c$  is a column of  $\mathbf{M}_N$ . Since  $\Omega_+(G_1, c) = \Omega_-(G_1, c) + 1$ , by (3) we thus have  $F(G_1, c) = 1$ . ■

*Lemma 2:* Let  $G_1 \subset G$  where  $|G_1| = g$  and  $g$  is a nonzero even integer  $\leq N - 1$ . There exists a column  $c$  in  $M_N$  such that  $F(G_1, c) = 0$ .

*Proof:* Let  $G_1 \subset G$  where  $|G_1| = g$  and  $g$  is a nonzero even integer. We create a column  $c$  such that  $\Omega_+(G_1, c) = \Omega_-(G_1, c) = \frac{g}{2}$ ,  $\Omega_+(G'_1, c) = \frac{N-g+1}{2}$ , and  $\Omega_-(G'_1, c) = \frac{N-g-1}{2}$ . Since  $G = G_1 \cup G'_1$  and  $G_1 \cap G'_1 = \emptyset$ , we yield

$$\begin{aligned}\Omega_+(G, c) &= \Omega_+(G_1, c) + \Omega_+(G'_1, c) = \frac{N+1}{2} = R \\ \Omega_-(G, c) &= \Omega_-(G_1, c) + \Omega_-(G'_1, c) = \frac{N-1}{2} = R - 1\end{aligned}$$

which shows that  $c$  is a column of  $M_N$ . Since  $\Omega_+(G_1, c) = \Omega_-(G_1, c)$ , by (3) we thus have  $F(G_1, c) = 0$ . ■

*Proposition 1:* Consider a certain system with  $N$  stations where  $N \geq 1$ . For any  $G_1, G_2 \subset G$  and  $G_1, G_2 \neq \emptyset$ ,  $\vec{G}_1 = \vec{G}_2$  iff  $G_1 = G_2$ .

*Proof:* By definition given in (4), we can easily see that if  $G_1 = G_2$ , then  $\vec{G}_1 = \vec{G}_2$ . In the following, we shall prove that if  $G_1 \neq G_2$ , then  $\vec{G}_1 \neq \vec{G}_2$ .

We will examine two cases: (i)  $|G_1|$  is an odd integer; (ii)  $|G_1|$  is an even integer. Without loss of generality, we assume that  $|G_1| \geq |G_2|$ .

Case (i):  $|G_1|$  is an odd integer. Let  $g = |G_1|$ . Based on lemma 1, there exists a column  $c$  in  $M_N$  such that

$$F(G_1, c) = 1, \quad (6)$$

and hence according to (4), we also have

$$\vec{G}_1(c) = 1.$$

This column will have the following property

$$\begin{aligned}\Omega_+(G_1, c) &= \frac{g+1}{2} \\ \Omega_-(G_1, c) &= \Omega_+(G_1, c) - 1 = \frac{g-1}{2} \\ \Omega_+(G'_1, c) &= \Omega_-(G'_1, c) = \frac{N-g}{2}\end{aligned} \quad (7)$$

with any permutation.

If  $G_1 \neq G_2$ , there exists  $K_1 = G_1 \setminus G_2 \neq \emptyset$  and  $K_2 = G_2 \setminus G_1$  where  $|K_1| = k_1 \geq 1$  and  $|K_2| = k_2 \geq 0$ . It is clear that  $K_1 \subseteq G_1$ ,  $K_2 \subseteq G'_1$  imply

$$1 \leq k_1 \leq g, k_2 \leq N - g.$$

where  $g$  is an odd integer and  $N - g$  is an even integer.

With the above condition, there exists a permutation within  $c$  such that

$$\begin{aligned}\Omega_+(K_1, c) &= \lfloor \frac{k_1}{2} \rfloor + 1 \leq \frac{g+1}{2} \\ \Omega_-(K_1, c) &= \lfloor \frac{k_1-1}{2} \rfloor \leq \frac{g-1}{2} \\ \Omega_+(K_2, c) &= \lfloor \frac{k_2}{2} \rfloor \leq \frac{N-g}{2} \\ \Omega_-(K_2, c) &= \lfloor \frac{k_2+1}{2} \rfloor \leq \frac{N-g}{2}\end{aligned}$$

that confirms  $K_1 \subseteq G_1$ ,  $K_2 \subseteq G'_1$  given (7). The immediate result gives

$$F(K_1, c) = \begin{cases} 1, & k_1 = 1, 3, \dots \\ 2, & k_1 = 2, 4, \dots \end{cases} \quad (8)$$

and

$$F(K_2, c) = \begin{cases} -1, & k_2 = 1, 3, \dots \\ 0, & k_2 = 0, 2, 4, \dots \end{cases} \quad (9)$$

which yield

$$F(K_1, c) - F(K_2, c) \geq 1. \quad (10)$$

Since  $G_2 = (G_1 \setminus K_1) \cup K_2$ , with (6) and (10), we obtain

$$F(G_2, c) = F(G_1, c) - F(K_1, c) + F(K_2, c) \leq 0$$

and hence  $\vec{G}_2(c) = 0$ . Since  $\vec{G}_1(c) = 1$ , there exists a column in  $M_N$  such that  $\vec{G}_2(c) \neq \vec{G}_1(c)$ , which is sufficient to show that  $\vec{G}_1 \neq \vec{G}_2$ .

Case (ii):  $|G_1|$  is an even integer. Let  $g = |G_1|$ . Based on lemma 2, there exists a column  $c$  in  $M_N$  such that

$$F(G_1, c) = 0 \quad (11)$$

and hence according to (4), we also have

$$\vec{G}_1(c) = 0.$$

This column will have the following property

$$\begin{aligned}\Omega_+(G_1, c) &= \Omega_-(G_1, c) = \frac{g}{2} \\ \Omega_+(G'_1, c) &= \frac{N-g+1}{2} \\ \Omega_-(G'_1, c) &= \frac{N-g-1}{2}\end{aligned} \quad (12)$$

with any permutation.

If  $G_1 \neq G_2$ , there exist  $K_1 = G_1 \setminus G_2 \neq \emptyset$  and  $K_2 = G_2 \setminus G_1$  where  $|K_1| = k_1 \geq 1$  and  $|K_2| = k_2 \geq 0$ . It is clear that  $K_1 \subseteq G_1$ ,  $K_2 \subseteq G'_1$  imply

$$1 \leq k_1 \leq g, k_2 \leq N - g.$$

where  $g$  is a nonzero even integer and  $N - g$  is an odd integer.

We first exclude the condition where  $k_2 = 0$ . With the above condition, there exists a permutation within  $c$  such that

$$\begin{aligned}\Omega_+(K_1, c) &= \lfloor \frac{k_1}{2} \rfloor \leq \frac{g}{2} \\ \Omega_-(K_1, c) &= \lfloor \frac{k_1-1}{2} \rfloor \leq \frac{g}{2} \\ \Omega_+(K_2, c) &= \lfloor \frac{k_2}{2} \rfloor + 1 \leq \frac{N-g+1}{2} \\ \Omega_-(K_2, c) &= \lfloor \frac{k_2-1}{2} \rfloor \leq \frac{N-g-1}{2}\end{aligned}$$

that confirms  $K_1 \subseteq G_1$ ,  $K_2 \subseteq G'_1$  given (12). The immediate result gives

$$F(K_1, c) = \begin{cases} -1, & k_1 = 1, 3, \dots \\ 0, & k_1 = 2, 4, \dots \end{cases} \quad (13)$$

and

$$F(K_2, c) = \begin{cases} 1, & k_2 = 1, 3, \dots \\ 2, & k_2 = 2, 4, \dots \end{cases} \quad (14)$$

which yield

$$F(K_2, c) - F(K_1, c) \geq 1. \quad (15)$$

For  $k_2 = 0$  which implies  $G_2 \subset G_1$ , since  $G_2$  cannot be empty,  $k_1 = g - |G_2|$  must be  $\leq g$ , there exists another permutation in  $c$  such that  $\Omega_+(K_1, c) < \Omega_-(K_1, c) \leq \Omega_-(G_1, c)$  giving

$$F(K_1, c) = \begin{cases} -1, & k_1 = 1, 3, \dots \\ -2, & k_1 = 2, 4, \dots \end{cases} \quad (16)$$

and  $F(K_2, c) - F(K_1, c) \geq 1$  as in (15) since  $F(K_2, c) = 0$ .

Given that  $G_2 = (G_1 \setminus K_1) \cup K_2$ , with (11) and (15), we obtain

$$F(G_2, c) = F(G_1, c) - F(K_1, c) + F(K_2, c) \geq 1$$

and hence  $\overrightarrow{G_2}(c) = 1$ . Since  $\overrightarrow{G_1}(c) = 0$ , there exists a column in  $\overrightarrow{M_N}$  such that  $\overrightarrow{G_2}(c) \neq \overrightarrow{G_1}(c)$  which is sufficient to show that  $\overrightarrow{G_1} \neq \overrightarrow{G_2}$ .

With the above two cases, we have proven that if  $G_1 \neq G_2$ , then  $\overrightarrow{G_1} \neq \overrightarrow{G_2}$ . Together with the earlier establishment that if  $G_1 = G_2$ , then  $\overrightarrow{G_1} = \overrightarrow{G_2}$ , we conclude that for any  $G_1, G_2 \subset G$  and  $G_1, G_2 \neq \emptyset$ ,  $\overrightarrow{G_1} = \overrightarrow{G_2}$  iff  $G_1 = G_2$ . ■

With Proposition 1, we have shown that based on our coding scheme, a particular decoded bitstream uniquely identifies a particular combination of receivers. This further allows the decoder to identify the presence of each individual receiver in the superimposed transmission. In the case that no receiver replies the acknowledgement, no transmission will occur on the channel which indicates the absence of all receivers.

#### IV. CONCLUSION

Motivated by the concept of PNC and the idea of decoding ACKs for improved broadcast reliability, we addressed the weaknesses of the current designs and proposed a coding scheme that achieves robust collision decoding without the need for precise energy detection. We first established the majority principle between the input and the output bitstreams based on BPSK modulation. Using the majority principle, we developed a coding scheme achieving the identification of individual acknowledgement transmission from a collided transmission. We proved that our proposed coding scheme guarantees the uniqueness of the decoded bitstream for a particular receiver combination.

While our given approach has been demonstrated for BPSK modulation scheme, it can similarly be extended to other modulation schemes which satisfy the general PNC modulation-demodulation mapping principle. Hence it is possible to implement a modified collision coding schemes for M-ary PSK, PAM, M-ary QAM, (M-ary represent the set of digital symbols) as well, which satisfy the general PNC modulation-demodulation mapping principle. We expect this to be part of our future research work.

#### REFERENCES

- [1] S. Zhang, S. C. Liew, and P. P. Lam, "Hot topic: physical-layer network coding," In ACM MobiCom 2006, Los Angeles, USA, Sept 2006.
- [2] K. Lu, S. Fu, Y. Qian, H.-H. Chen, "On Capacity of Random Wireless Networks with Physical-Layer Network Coding," In IEEE Journal on Selected Areas of Communications, Vol 27, Issue 5, p 763 - 772, Jun 2009.
- [3] M. Durvy, C. Fragouli, and P. Thiran, "Towards Reliable Broadcasting using ACKs," In IEEE ISIT 2007, Nice, France, June 2007.
- [4] E. Pagani and G. P. Rossi, "Reliable broadcast in mobile multihop packet network," In ACM MobiCom, Budapest, Hungary, Sept 1997.
- [5] K. Tang and M. Gerla, "MAC reliable broadcast in ad hoc networks", In IEEE MILCOM, Vienna, USA, Oct 2001.
- [6] R. Ahlswede, N. Cai, S.-Y. R. Li, and R. W. Yeung, "Network Information Flow, In IEEE Transactions on Information Theory, Vol 46, Issue 4, July 2000.
- [7] S. Katti, H. Rahul, W. Hu, D. Katabi, M. Medard, and J. Crowcroft, "XORs in the Air: Practical Wireless Network Coding," In IEEE/ACM Transactions on Networking, Vol 16, Issue 3, p 497 - 510, Jun 2008.