Comments on IEEE 802.11 Saturation Throughput Analysis with Freezing of Backoff Counters

Chuan Heng Foh, *Member, IEEE*, and Juki Wirawan Tantra, *Student Member, IEEE*

***Abstract*— This letter presents an accurate model for the performance analysis of the IEEE 802.11 saturation throughput with freezing of the backoff counter. The model corrects the existing model presented by Ziouva and Antonakopoulos which assumes that the channel access probability and station collision probability are independent of channel status. Simulation results show the accuracy of the new model.**

***Index Terms*— Wireless local area networks, performance analysis, saturation throughput analysis.**

I. Introduction

THE bi-dimensional Markov Chain modeling introduced by Bianchi [1] for the analysis of the IEEE 802.11 saturation throughput has become a common method to study the performance of the IEEE 802.11 Medium Control Access (MAC) protocol [2] and its enhancements. The model was later refined to capture further details of the IEEE 802.11 protocol operations. Among the refinements, one is due to Ziouva and Antonakopoulos [3] aiming to capture the freezing of backoff counters when the broadcast channel is sensed busy by a station. Precisely, when a channel turns idle from busy due to, for example, a Distributed InterFrame Space (DIFS), Bianchi's model assumes that each station immediately reactivates and decrements its counter, whereas the IEEE 802.11 standard specifies that a backoff counter is decremented only after the channel continues to remain idle for a predefined slot time. The refinement reported in [3] was, however, introduced without realizing that the two key probabilities governing the performance, namely the channel access probability, τ, and the station collision probability, p, depend on the channel status. This inaccuracy in the model affects several important measures including the probabilities that a particular time period on the broadcast channel is an idle, a successful transmission, or a transmission collision period. Owing to the large difference in the duration of the different types of time periods, the inaccuracy does not reflect significantly in the final saturation throughput results in most common cases; however, fundamentally, the model is in error.

In this letter, we present a new model correcting that of [3] by evaluating the channel access probabilities and the station collision probabilities conditioned upon the channel status. We then show the accuracy of our results via computer simulation, and demonstrate the errors if such details are ignored.

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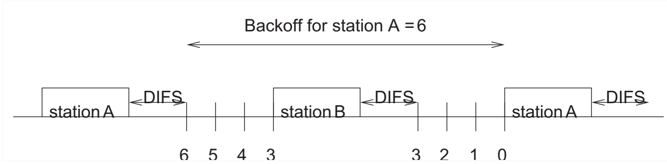

Fig. 1. IEEE 802.11 basic access method.

II. Saturation Throughput Analysis

The mechanism of the IEEE 802.11 Distributed Coordination Function (DCF) with basic access method is shown in Fig. 1. It differs from the model presented in [1] in the decrement of the backoff counter. The IEEE 802.11 standard [2] specifies that a station freezes its backoff counter when it detects a transmission on the channel (note that the backoff counter is not decreased during the channel busy period). This backoff freezing procedure directly affects the probability that a station accesses the channel, and this probability also depends on whether the previous period is busy or idle.

To understand this, we first consider the channel access event after a busy period due to a collision. After a collision, since stations that did not participate in this collision had frozen their backoff counters, they will not access the channel after the busy period; only those suffered a collision may access the channel if their newly chosen backoff counter is zero. Hence, only a group of stations rather than all stations may access the channel after a busy period. In case of a successful transmission, only one station, which performed the successful transmission, may access the channel after the successful transmission period. As opposed to the case of a busy period, after an idle period, all stations whose backoff counters are decremented to zero will access the channel; hence, it is obvious that the channel access probability actually depends on whether the previous period is idle or busy. This is not modeled in [3] where the derived channel access probability is not conditioned upon the type of the previous time period.

The new model describing the backoff process of a station for the saturation network condition is presented in Fig. 2. Here we consider a network of n saturated stations. The state $\{i, j, k\}$ represents the state of a station at a particular time period, where i indicates the type of the previous period, either idle or busy ($i = 0$ or $i = 1$ respectively); j indicates the current backoff stage ($j = 0, 1, \ldots, m$); and k indicates the current backoff counter ($k = 0, 1, \ldots, W_j - 1$). Two important protocol parameters describing the backoff process are the minimum backoff window denoted by W_0, and the maximum backoff stage denoted by m. The backoff window of a station

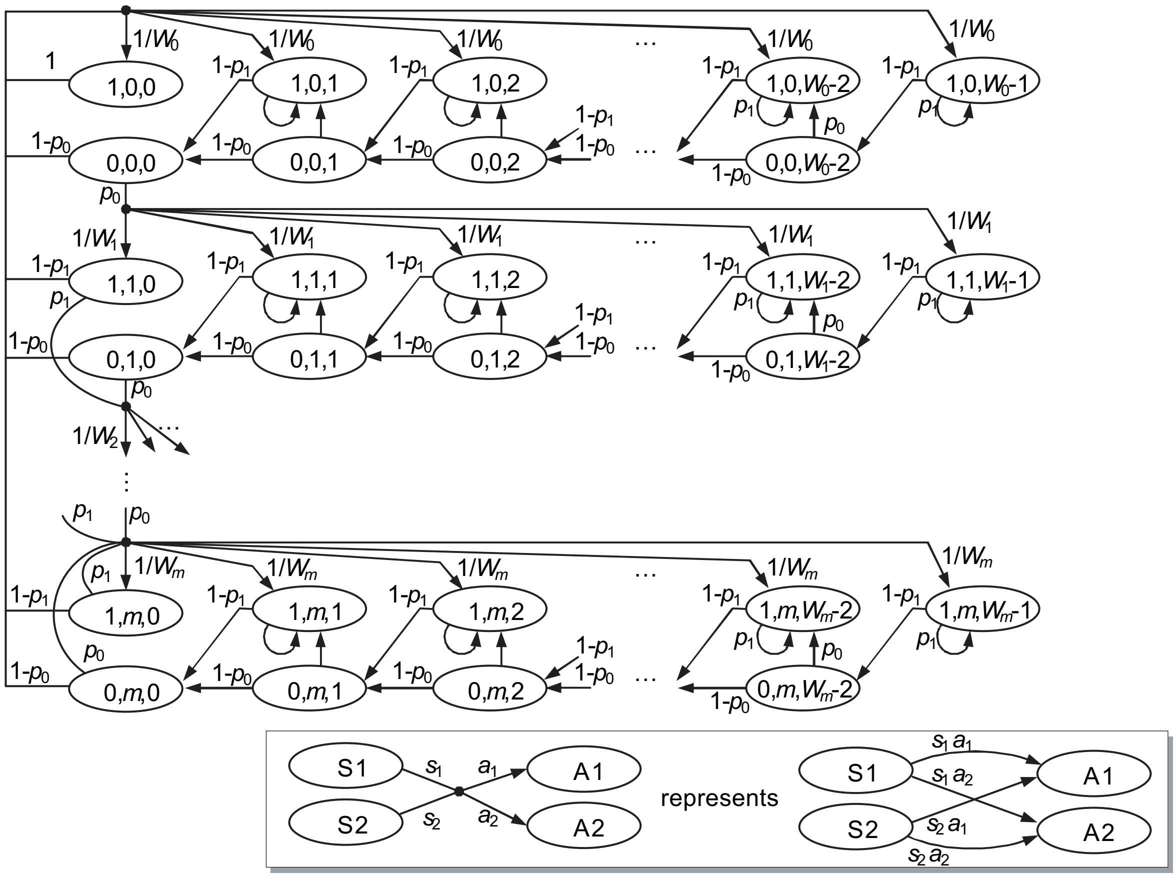

Fig. 2. The Markov chain representation of the new model.

at the j-th backoff stage is W_j, or precisely $2^j W_0$.

Two key probabilities governing the backoff process are first defined, they are, p_0 (p_1), the probability that, from a station's point of view, at least one of the other stations transmit during a slot after an idle (a busy) period. We further define q_0 (q_1) to be the probability that the broadcast channel remains idle after an idle (a busy) period.

Given that the channel is either an idle or a busy period, let P_i be the probability that a particular period on the channel is idle, then $P_i = q_0 P_i + q_1(1 - P_i)$, which gives

$$P_i = \frac{q_1}{1 - q_0 + q_1}. \tag{1}$$

Let $b_{i,j,k}$ be the stationary distribution of the described Markov Chain. Each of the state stationary probability can be expressed in terms of $b_{1,0,0}$ as

$$\begin{aligned} b_{1,j,0} &= \psi_j b_{1,0,0} \qquad \text{for } j = 1, 2, \ldots, m, \\ b_{1,j,k} &= \frac{1 + p_0(W_j - 1 - k)}{1 - p_1} \psi_j b_{1,0,0} \\ &\quad \text{for } k = 1, 2, \ldots, W_j - 1 \text{ and } j = 0, 1, \ldots, m, \\ b_{0,j,k} &= (W_j - 1 - k)\psi_j b_{1,0,0} \\ &\quad \text{for } k = 0, 1, \ldots, W_j - 2 \text{ and } j = 0, 1, \ldots, m, \end{aligned} \tag{2}$$

where

$$\psi_j = \begin{cases} 1 & \text{if } j = 0, \\ \dfrac{p_0(W_0 - 1)}{W_1} & \text{if } j = 1, \\ \dfrac{p_0(W_0 - 1)}{W_1}\pi_j & \text{if } j = 2, 3, \ldots, m-1, \\ \dfrac{p_0(W_0 - 1)}{W_1}\pi_j \dfrac{W_m}{W_m - p_1 - p_0(W_m - 1)} & \text{if } j = m, \end{cases}$$

and

$$\pi_j = \prod_{x=2}^{j} \left[\frac{p_1}{W_x} + \frac{p_0}{W_x}(W_{x-1} - 1) \right]$$

with

$$\sum_{j=0}^{m} \left[\sum_{k=0}^{W_j - 2} b_{0,j,k} + \sum_{k=0}^{W_j - 1} b_{1,j,k} \right] = 1.$$

Define τ_i and τ_b to be the probabilities that a station accesses the broadcast channel after an idle and a busy period respectively. Similar to [3] and [1], each of τ_i and τ_b can be expressed as a function of the stationary probabilities. They are given by

$$\begin{aligned} \tau_i &= \frac{\sum_{j=0}^{m} b_{0,j,0}}{\frac{q_1}{1 - q_0 + q_1}}, \\ \tau_b &= \frac{\sum_{j=0}^{m} b_{1,j,0}}{1 - \frac{q_1}{1 - q_0 + q_1}}. \end{aligned} \tag{3}$$

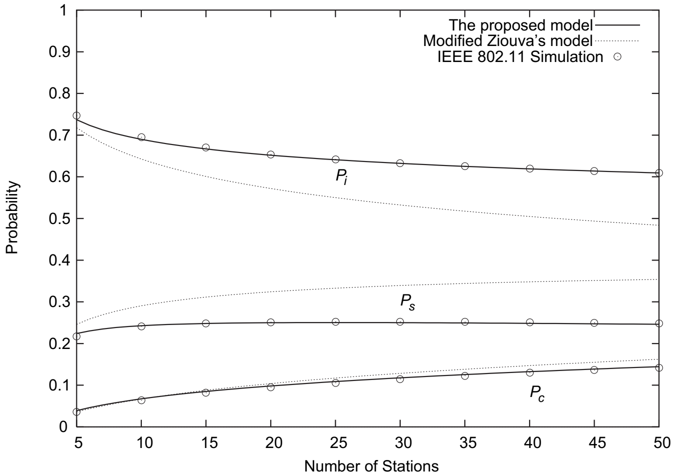

Fig. 3. Probabilities of an idle, a successful transmission, and a collision of various models for $W = 16$ and $m = 6$.

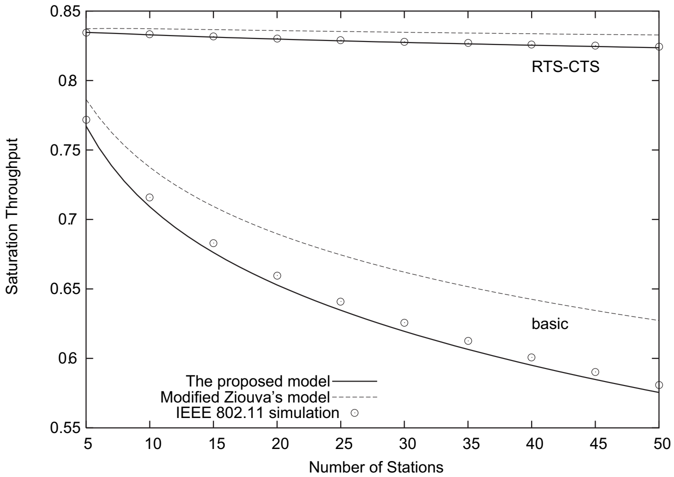

Fig. 4. Saturation throughput of various models for $W = 16$ and $m = 6$.

Having obtained τ_i and τ_b, q_0 (q_1) can be determined based on the fact that the broadcast channel remains idle after an idle (a busy) period if no station accesses the channel. Thus we get

$$\begin{aligned} q_0 &= (1 - \tau_i)^n, \\ q_1 &= (1 - \tau_b)^n. \end{aligned} \tag{4}$$

A particular station finds a slot to be busy if at least one of the other stations access the channel. Hence we have

$$\begin{aligned} p_0 &= 1 - (1 - \tau_i)^{n-1}, \\ p_1 &= 1 - (1 - \tau_b)^{n-1}. \end{aligned} \tag{5}$$

The system throughput S, the fraction of time used for successful payload transmission, can be expressed as

$$S = \frac{P_s E[P]}{P_i \sigma + P_s T_s + P_c T_c}, \tag{6}$$

where $E[P]$ is the average payload length, σ is the duration of an empty slot time, T_s is the average time that the channel is sensed busy because of a successful transmission, and T_c is the average time that the channel is sensed busy due to a collision. These quantities for the basic and the RTS-CTS access methods are given by (14) and (17) in [1].

The probabilities P_i, P_s and P_c describe the probabilities that a particular period on the channel carries no transmission (idle), a successful data frame transmission, and two or more transmissions (collision), respectively. Probability P_i is given by (1) and probabilities P_s and P_c can be expressed as

$$\begin{aligned} P_s &= n\tau_i(1 - \tau_i)^{n-1} P_i + n\tau_b(1 - \tau_b)^{n-1}(1 - P_i), \\ P_c &= 1 - P_i - P_s. \end{aligned} \tag{7}$$

III. Numerical Results

In Figs. 3 and 4, numerical results for the saturation throughput along with several important probabilities obtained from our model are plotted and compared with that of [3][1]. We use the same protocol parameters as [1] for this comparison.

The numerical results are obtained using the fixed point iteration technique [5]. In brief, initial guessing for $\tau_i^{(0)}$ and $\tau_b^{(0)}$ were first made, then p_0 and p_1 were computed by (5), and later used to calculate $b_{i,j,k}$ using (2). After that, new values for τ_i^* and τ_b^* were obtained by (3) and (4). We finally updated $\tau_i^{(1)}$ and $\tau_b^{(1)}$ for the next iteration by $\tau_i^{(1)} = 0.5\tau_i^{(0)} + 0.5\tau_i^*$ and $\tau_b^{(1)} = 0.5\tau_b^{(0)} + 0.5\tau_b^*$. The simulation results (shown with symbols) are obtained with a 95% confidence interval lower than 0.001. As in [1], our model assumes that after the completion of a transmission, each station detects a collision after a DIFS period. This makes the collided stations synchronized to other stations. This assumption is relaxed in the simulation, where we consider each station reveals a collision after the acknowledgment timeout expires, which is $0.3\ ms$.

As can be seen in Figs. 3 and 4, our model gives accurate results for the performance of IEEE 802.11 with freezing of backoff counters. For the model of [3], only small errors in saturation throughput are produced for the case of the RTS-CTS method as data frame transmission duration is significantly longer than other time periods, where the model of [3] appears to be accurate if not carefully studied.

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[1] Two modifications were applied to the original model, which are (i) the special state, $\{-1, 0\}$, modeled in [3] is said to be inconsistence with the standard [4], it was removed; and (ii) equation (8) in [3] is corrected as follows: $p_b = 1 - (1 - \tau)^{n-1}$.